THE HAWORTH SERIES

NUMBER APPLICATIONS

SCHUYLER
AND
VAN
SICKLE

NUMBER APPLICATIONS

THE HAWORTH SERIES OF ARITHMETICS

NUMBER APPLICATIONS

BY

ERWIN H. SCHUYLER

INSTRUCTOR IN MATHEMATICS IN THE SACHS COLLEGIATE INSTITUTE FOR BOYS, NEW YORK

AND (AS CRITICAL ADVISER)

JAMES H. VAN SICKLE

SUPERINTENDENT OF CITY SCHOOLS, BALTIMORE, MD.

NEW YORK
NEWSON AND COMPANY, PUBLISHERS
1906

Norwood Press
J. S. Cushing & Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

PREFACE

THE HAWORTH SERIES OF ARITHMETICS consists of three books, "Number Foundations," for primary grades, "Number Relations," for intermediate grades, and "Number Applications," for grammar grades. "Number Foundations" is divided into two parts.

The first part is intended to cover the work of the first school year. It need not be in the hands of the pupils, but will serve as a guide to the teacher in planning oral lessons. The second part, published for pupils' use as a separate volume, covers the work of the second or third school year.

The course is so arranged that it can be adapted readily to local conditions, such as the maturity of the pupils, length of the school year, or the requirements of the course of study.

Under favorable conditions, the series may be finished at least one year earlier than the arithmetics of the older type. This is made possible by the elimination of obsolete and useless material, and by careful gradation.

The series is the outcome of long experience in teaching and supervision. In preparing it the authors have aimed first of all to give the pupil a knowledge of the essentials of the subject and independent power of application without consuming an undue portion of his time or that of his teacher. They realize that other subjects in the curriculum, rich in educative value, have too commonly failed to receive their fair share of attention on account of the excessive demands of arithmetic.

The old arithmetics were unnecessarily difficult; partly

because the order followed in presenting material failed to conform to the order in which the child naturally acquires mathematical ideas; partly because of the many obsolete and useless topics treated; and partly because of the puzzling problems introduced solely for the sake of the mental discipline which their solution was said to afford.

The method of presentation here followed is closely inductive. Problems are not assembled promiscuously or by chance. In general, each lesson presents a development in which every problem is seen to be a necessary step toward a generalization or the mastery of a process. In like manner each lesson usually serves as a direct preparation for the next lesson. The careful attention given to grading has resulted in a set of books that the pupil can use by himself. Difficulties there are, but these have been led up to in such a way that the pupil is not forced to rely constantly upon the teacher for guidance and help.

Drill exercises have been freely introduced to fix processes that have been learned. Set reviews have, for the most part, given place to exercises calling for the application of what has been learned to new conditions.

The criterion of use has been the one adopted in choosing the topics treated. The fundamental operations of arithmetic with their simple applications are the only ones that pupils in elementary schools have opportunity to employ. After developing these and fixing their technique by drill, continued use of arithmetic in connection with drawing, manual training, geography, physics, etc., as called for in considering the quantitative aspect of such studies, will furnish sufficient practice to keep arithmetic a ready tool. The time allotted to mathematics in the seventh and eighth school years may with advantage be devoted largely to further study of constructive geometry and algebra.

In the first two books the pupil is led, by means of exercises involving small numbers, through a wide range of arithmetical experience; then, with somewhat larger numbers and greater complications, he again and again encounters familiar types which he gradually comes to recognize as separate arithmetical topics, but he is never allowed to lose sight of the connections between these topics. After a fair idea of the field of arithmetic has been gained and a sense of its usefulness developed, the distinctly topical plan of treatment is begun. In the third book each large topic is sufficiently isolated to permit its measurably complete development.

Believing that there is a technique that must be mastered, and that this mastery comes through concentration upon the thing in hand, the authors have not attempted to teach arithmetic incidentally. They have sought problems that would emphasize the use of what is learned, but they have not, as some advocate doing, developed arithmetical topics through the study of industries or geographical and scientific material as such. They believe that to do so is to violate the true principle of correlation, which they understand to be this: "In giving a lesson or a series of lessons upon any particular topic, the teacher should press into his service allied material *that will help toward a completer grasp of the topic under consideration*, but should exclude all else."

Applying this principle, they would assent to the proposition that arithmetic may and should serve a valuable purpose in the quantitative study of topics, say, in commercial or industrial geography; but this is merely saying that arithmetic should be used in such branches to the extent that it serves this purpose. It is by no means equivalent to saying that a textbook in arithmetic should be written with the divided aim of teaching arithmetic and geography, or arithmetic and farming,

or arithmetic and any other subject. True correlation in any subject requires that secondary topics shall assist toward a completer understanding of the primary topics. In a text-book in arithmetic the primary topics are manifestly arithmetical ones, and all other topics are secondary.

Arithmetic, like other studies, possesses a unity and continuity of its own which cannot be freely violated without a wasteful scattering of effort. We must follow the order dictated by the nature of the subject and by the pupil's stage of mental development, taking advantage, however, of the multitudinous opportunities that will arise of really helpful correlation. The teacher can select allied material from the child's experience or environment, from current industrial or business life, from science or nature study, material which in its nature is entirely local or transient. While it is desirable to draw upon familiar occupations for problems, it is not true economy of the pupil's time to make an arithmetic a cyclopedia of useful information.

Furthermore, attempts which have been made to frame the problems of a lesson or a series of lessons in arithmetic in such a way as to present a study of an occupation are for the most part open to the objection that the brevity of treatment necessary in an arithmetic defeats the informational aim, and that the topic as an industry demands greater maturity and larger experience on the part of the pupil than do the arithmetical computations really necessary to the study. Either the large industrial topic is too difficult to be timely or the arithmetic is too easy. In this respect, also, we see that the true principle of correlation is violated, since the study of the industry brings no proportionate increase of arithmetical knowledge.

It will be observed that some topics usually treated late in the course appear much earlier in this series of books. This

is made possible by the omission of much work that used to be given under the head of compound denominate numbers and other topics of comparatively little use and by the careful way in which difficulties are approached. The time limit as applied in addition, the graphic method of acquainting pupils with the facts of multiplication and division through their own efforts before the tables are required, calculations from drawings made to a scale, and the graphic treatment of fractions and ratio are among the devices employed in these books to stimulate interest, to vivify ideas, and to minimize drudgery.

Such topics as insurance, taxes, discounting at banks, square root, etc., need not be deferred on account of their difficulty if presented through simple problems, as in the third book of this series.

Geometric forms have been freely introduced from the beginning. By the time pupils reach the more difficult parts of mensuration they are already familiar with many of the forms and understand area and volume through much previous practice. In the second book algebraic symbols are introduced, and in the third book continued in such a way as to acquaint the pupil with some of their uses and to introduce him to the study of algebra. No work with geometric forms or algebraic symbols has been introduced which has not been as fully tested in the class room as the more ordinary kinds of problems.

CONTENTS

NUMBER APPLICATIONS

I. NOTATION

(For reference.*)

1. The *Arabic Notation* employs the ten characters, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, in writing numbers. The first nine are called *significant* figures, because each expresses a value when standing alone. The last one is called *naught*, *cipher*, or *zero*.

2. The *Roman Notation* employs the seven capital letters, I, V, X, L, C, D, M, in writing numbers. Occasionally small letters are used instead of capitals.

I = 1, V = 5, X = 10, L = 50, C = 100, D = 500, M = 1000.

(*a*) When a letter is repeated, its value is repeated.

I = 1, II = 2, III = 3. C = 100, CC = 200, CCC = 300.

(*b*) When a letter is annexed to another of greater value, the two represent the sum of their separate values.

VI = 6, XI = 11, CI = 101, MV = 1005.

(*c*) When a letter is prefixed to another of greater value, the two represent the difference of their separate values. IV = 4, IX = 9, IC = 99, VM = 995.

* Pupils should be carefully shown by the teacher how to gain knowledge of the reference matter here and elsewhere by using it in working out the exercises instead of memorizing it.

(*d*) When a short dash is placed over a letter, the letter represents a value one thousand times its value alone.

$$\overline{V} = 5000, \quad \overline{C} = 100{,}000, \quad \overline{D} = 500{,}000.$$

3. Write nine in both ways.
4. Write ninety in both ways.
5. Write ninety-six in both ways.
6. Write nine hundred ninety-six in both ways.
7. Write 1904 in the Roman System.
8. Write MDCCCXCVII in the Arabic System.
9. Write MCMIII in the Arabic System.

II. NOTATION (Continued)

1. Write one thousand eighty-four in both systems.
2. Write six hundred sixty-four in both systems.
3. Write seven hundred seventy-seven in both systems.
4. Write 1776 in the Roman System.
5. Write 1812 in the Roman System.
6. Write 1825 in the Roman System.
7. Write MDCCCI in the Arabic System.
8. Write MDCCCLXI in the Arabic System.
9. Write MDCCCXCIX in the Arabic System.
10. Write 1783 in the Roman System.
11. Write 1763 in the Roman System.

12. Write MCMIV in the Arabic System.

13. Write MDCCXCIX in the Arabic System.

NOTE.—The *Roman Notation* is used less now than formerly. It is used in numbering the chapters of books, the hours on watches and clocks, and in expressing the number of the year in inscriptions on public buildings and memorial structures.

III. ORDERS OF UNITS. PERIODS

(For reference.)

Units of the 12th order.	Units of the 11th order.	Units of the 10th order.	Units of the 9th order.	Units of the 8th order.	Units of the 7th order.	Units of the 6th order.	Units of the 5th order.	Units of the 4th order.	Units of the 3d order.	Units of the 2d order.	Units of the 1st order.		Units of the 1st decimal order.	Units of the 2d decimal order.	Units of the 3d decimal order.	Units of the 4th decimal order.	Units of the 5th decimal order.	Units of the 6th decimal order.	Units of the 7th decimal order.
5	6	7	8	4	0	3	7	5	6	3	4	.	9	5	6	3	1	0	7

567,840,375,634.9,563,107

Beginning at the right, point off the decimal into as many periods as possible of three figures each. Then continuing from the decimal point, point off the whole numbers into as many periods as possible of three figures each.

The periods are named thus:

Billion	Million	Thousand	Primary Units				
567,	840,	375,	634	.	9,	563,	107

This number is read thus: five hundred sixty-seven billion eight hundred forty million three hundred seventy-

five thousand six hundred thirty-four and nine million five hundred sixty-three thousand one hundred seven ten-millionths.

Bear in mind that the word *and* is used only at the decimal point. In reading the whole number, the words *primary units* may be used upon reading the last period, but their use is not necessary, the name being understood. The numerator of the decimal, being a whole number, is read as a whole number, the denominator, ten million, giving the name ten-millionths to the decimal.*

Hundreds of billions.	Tens of billions.	Billions.	Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Primary units.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.
5	6	7,	4	8	0,	3	7	5,	6	3	4 .	9,	6	5	3,	1	0	7

In the above representation each order of units has been designated by its appropriate name.

4.8 is read four and eight tenths; or 4 primary units and 8 tenths. Sometimes it may be desirable to read it 48 tenths.

239.49 is read two hundred thirty-nine and forty-nine hundredths. Do not say two hundred *and* thirty-nine. The *and* should be used only where the decimal point occurs.

Each significant figure represents one or more units, the kind of units depending upon its place in the number. Thus, 5 standing alone represents a number of primary

* The denominator of a decimal may always be expressed by 1 with as many ciphers annexed as there are places in the decimal.

units. In the number 50, 5 represents the same number of units as before, but each unit is ten times as large as before. In the number 500, 5 represents the same number of units still, but each unit is one hundred times as large as in the first instance and ten times as large as in the second. In short, the units of any order are ten times as large as those of the next lower order.

The naught, cipher, or zero is used to mark the absence of units in any order. As in 306, there are 3 hundreds, no tens, and 6.

NOTE. — The periods above billion are: trillion, quadrillion, quintillion, sextillion, septillion, octillion, nonillion, decillion, undecillion, duodecillion, tredecillion, quatuordecillion, quindecillion, sexdecillion, septemdecillion, octodecillion, novemdecillion, vigintillion.

IV. READING DECIMALS (*a*)

4578.059 may be read: (1) 4 thousands, 5 hundreds, 7 tens, 8 primary units, no tenths, 5 hundredths, 9 thousandths; (2) four thousand five hundred seventy-eight and fifty-nine thousandths.

Read both ways (avoid the word *and* except at the decimal point):

1. 5243.076.	7. 4.376005.	13. 87654.23.
2. 3784.932.	8. 2.340056.	14. 9800.017.
3. 425.3084.	9. 379450.6.	15. 4900.068.
4. 734.5207.	10. 856700.3.	16. 70009.75.
5. 86.52304.	11. 92768.49.	17. 80002.04.
6. 52.83456.	12. 3640.903.	18. 3400.319.

V. READING DECIMALS (*b*)

(Thoroughness here will save trouble later.)

1. 345 = 3 hundreds + 4 tens + 5 primary units = 345 primary units.

2. 34.5 = 3 tens + 4 primary units + 5 tenths = 345 tenths.

3. 3.45 = 3 primary units + 4 tenths + 5 hundredths = 345 hundredths.

4. .0345 = 3 hundredths + 4 thousandths + 5 ten-thousandths = 345 ten-thousandths.

5. 5.67 = 5 primary units + 6 —— + 7 —— = 567 ——ths.

6. 2.39 = —— hundredths.

7. 3.072 = —— thousandths.

8. 8.54 = —— ——.

9. 6.057 = —— ——.

10. 5.3 = —— ——.

11. 62.5 = —— ——.

12. 54.67 = —— ——.

13. 725.18 = —— ——.

NOTE. — It is evident that the name of the units of the right-hand figure of a number may be applied to the entire number.

VI. READING DECIMALS (*c*)

1. 2.345 = 23 tenths and 45 thousandths.

2. 2.345 = 234 hundredths and 5 thousandths.

Read the following:

3. 42.63 = —— tenths and —— hundredths.

4. .2345 = —— hundredths and —— ten-thousandths.

5. .2345 = —— thousandths and —— ten-thousandths.

6. 726 = —— hundreds and —— primary units.

7. 726 = —— tens and —— primary units.

8. 5749 = —— thousands and —— primary units.

9. 5.749 = —— tenths and —— thousandths.

10. 83456 = —— hundreds and —— primary units.

11. 8.3456 = —— thousandths and —— ten-thousandths.

NOTE. — It is evident that a number may be read in parts by giving each part the name of the unit denoted by the last figure of that part.

VII. READING DECIMALS (*d*)

1. In reading .509, do *not* say five hundred *and* nine thousandths, but five hundred nine thousandths, leaving out the *and*.

2. Five hundred and nine thousandths is written thus: 500.009. This is also properly read, 500 primary units and 9 thousandths.

Read, using and *only at the decimal points:*

3. 400.007.*	9. 100.002.	15. 200.005.
4. .407.†	10. .102.	16. .205.
5. 200.003.	11. 100.003.	17. 300.008.
6. .203.	12. .103.	18. .308.
7. 800.006.	13. 100.004.	19. 500.06.
8. .806.	14. .104.	20. .506.

* Either 400 and 7 thousandths, pausing after saying 400; or 400 primary units and 7 thousandths.

† Four hundred seven thousandths.

VIII. WRITING DECIMALS

Write:

1. Two hundred and two hundredths.
2. Two hundred two hundredths.
3. Three thousand and three thousandths.
4. Three thousand three thousandths.
5. Five hundred and eight thousandths.
6. Five hundred eight thousandths.
7. Two hundred eight ten-thousandths.
8. One hundred nine ten-thousandths.
9. Five thousand three and six thousandths.
10. Two thousand four hundred seven and three thousand five hundred seven ten-thousandths.
11. Four million four thousand four and twenty-three thousandths.
12. Five million seventeen thousand fourteen and two thousand eighteen ten-thousandths.
13. Three million four and twenty-one thousandths.
14. One million twenty-seven and three tenths.

Read:

15. 100.001.	21. 700.001.	27. 1.013.
16. .101.	22. .701.	28. 23.2,005.
17. 900.008.	23. 7,007.004.	29. 175.8,095.
18. .908.	24. 20,002.01.	30. 83,976.005.
19. .800.005.	25. 3,003,003.03.	31. 9,003.1,008.
20. .805.	26. 90,000,009.43.	32. 2,002,000.10,026.

IX. ADDITION. SIMPLE NUMBERS

1. *Addition* is the process of finding one number equal to two or more numbers united.

2. The *addends* are the numbers added.

3. The *sum* is the answer.

4. The correctness of the work may be tested by adding the numbers again, but in the opposite direction.

Add 239, 678, and 297.

239
678
297
1214

The sum of the primary units is 24. 24 equals 2 units of the second order and 4 primary units. Write the 4 units of the first order, and add the two units of the second order to the like units in the second column.

The sum of the units of the second order is 21. This equals 2 units of the third order and 1 unit of the second order. Write the 1 unit of the second order and add the 2 units to the like units in the third column.

The sum of the units of the third order is 12. This equals 1 unit of the fourth order and 2 units of the third order, both of which we write in their proper places. The sum is 1214.

It will be observed that units of like order are in the same column.

Find the sum of:

1.	2.	3.	4.	5.
543	310	393	263	201
784	229	407	145	563
329	267	210	672	300
486	391	581	214	243
237	213	258	323	712
250	392	329	415	342
232	347	36	222	523
303	294	844	333	240
135	233	72	512	215

NOTE. — After a little practice any one of the preceding examples ought to be added correctly by a pupil in 25 seconds. Copy accurately, then time yourself.

X. ADDITION. DECIMALS (*a*)

Add:

$6.29	
27.64	
183.72	
19.27	
68.47	In adding long columns some such method as is here given is desirable, particularly in cases where the worker is liable to interruptions.
19.63	
75.84	
2.95	
95.04	
127.68	
42.65	
58	units of 2d decimal order.
56	units of 1st decimal order.
63	units of 1st integral order.
62	units of 2d integral order.
40	units of 3d integral order.
2	units of 4th integral order.
$6689.18	

Write in columns and add:

1. $3.27, $9.20, $7.29, $2.94, $.85, $7.90.
2. $29.05, $9.25, $9.08, $.73, $8.03, $8.97, $8.49.
3. $3.09, $5.75, $8.43, $9.28, $.47, $8.27.
4. $7.59, $6.45, $7.03, $6.78, $5.38, $6.77, $4.99.
5. $3.54, $4.63, $5.74, $6.59, $7.49, $9.62, $8.01.

Find the sum of:

6.	7.	8.	9.	10.
$5.60	$3.19	$3.93	$3.57	$4.07
6.89	2.27	6.05	4.83	4.12
6.32	2.10	.80	5.93	5.96
5.87	2.66	3.12	2.60	2.88
4.08	2.91	3.14	4.40	3.62
2.56	3.14	3.63	2.48	5.53
2.32	2.97	.37	2.32	2.00
3.03	2.90	2.88	2.48	3.41
1.72	2.48	2.48	3.41	.89

NOTE. — It does not require an expert accountant to find the *correct* sum in any one of the above examples in 20 seconds. These numbers were taken from a ledger.

XI. ADDITION. DECIMALS (*b*)

1. Add 25.6, 32.008, 19.34, 18.216.

2. Add 240.8, 9.016, 28.192, 27.009.

3. Add 1.8, 162.09, 17.04, 259.17.

4. Add 24, .016, 2.072, 18.025.

5. Add 246, 784, .019, 3.427.

6. Add 1.92, 17.8, 29.362, 8.45.

7. Add 63.2, 85.96, 27.016, 27.6.

8. Find the sum of three thousand and seven thousandths, and three thousand seven thousandths.

9. Find the sum of five hundred and six thousandths, and five hundred six thousandths.

10. Find the sum of twenty-one and six tenths, and twenty-six thousandths.

11. The sales of a grocery store were as follows: Monday, $13.72; Tuesday, $15.94; Wednesday, $14.96; Thursday, $16.40; Friday, $10.90; Saturday, $18.48. Find the sum of the sales for the week.

12. A boy picked raspberries for five days in succession, as follows: Tuesday, 23 quarts; Wednesday, 25 quarts; Thursday, 18 quarts; Friday, 28 quarts; Saturday, 24 quarts. How many quarts in all did he pick?

13. A farmer sold milk for one week, as follows: Sunday, 20.5 quarts; Monday, 18.5 quarts; Tuesday, 19 quarts; Wednesday, 21.5 quarts; Thursday, 20 quarts; Friday, 20 quarts; Saturday, 25.5 quarts. Find the total number of quarts.

14. Add $16.82, $54.06, $19.87, $49.72, $20.78.

XII. ADDITION. DENOMINATE NUMBERS (*a*)

Add 2 gal. 3 qt. 1 pt., 5 gal. 2 qt. 1 pt., 3 gal. 1 qt. 1 pt., and 4 gal. 3 qt. 1 pt.

gal.	qt.	pt.
2	3	1
5	2	1
3	1	1
4	3	1
16	3	0

Adding the pints, we obtain 4 pints. 4 pints equal 2 quarts. Adding the 2 quarts to the quarts in the next column, we obtain 11 quarts. 11 quarts equal 2 gallons and 3 quarts. Write the 3 quarts, and add the 2 gallons to the gallons in the next column. We obtain 16 gallons. The sum is 16 gal. 3 qt.

1. Read the first addend in the example just given.

2. Read the second addend.

3. How many addends are there?

4. The first addend is a —— denominate number.

5. Every addend in the example is a —— denominate number.

6. A compound denominate number is a number expressed in units of two or more denominations.

Add:

7.			8.			9.	
gal.	qt.	pt.	yd.	ft.	in.	mi.	rd.
5	3	1	4	2	10	48	119
4	2	1	3	1	11	63	115
6	1	1	7	0	9	21	260
7	2	1	6	2	8	56	29
5	3	1	9	1	6	72	316
9	1	1	4	1	7	94	49
22	0	1	5	2	5	18	235

Let the watchwords be accuracy and rapidity. Get the correct result at the *first* effort.

XIII. ADDITION. DENOMINATE NUMBERS (*b*)

The sign of addition (+) is read "plus."

The sign of equality (=) is read "equal," "equals," "is equal to," or "are equal to."

Read, filling in the blanks:

1. $6+7+5=$ ——.
2. $8+9+7=$ ——.
3. $7+5+3=$ ——.
4. $6+9+3=$ ——.
5. $3+4+8=$ ——.
6. $5+8+6=$ ——.
7. $3+6+7=$ ——.
8. $7+8+6=$ ——.
9. Copy Exs. 1–8, filling in the blanks.

Read, filling in the blanks:

10. 5 7's + 6 7's = —— 7's.
11. 3 6's + 8 6's = —— 6's.
12. 7 4's + 9 4's = —— 4's.
13. 6 9's + 9 9's = —— 9's.
14. 3 15's + 8 15's = —— 15's.
15. 4 3's + 9 3's = —— 3's.
16. 7 x's + 8 x's = —— x's.
17. 8 x's + 9 x's = —— x's.
18. 9 x's + 4 x's = —— x's.
19. 5 b's + 6 b's = —— b's.
20. 11 c's + 7 c's = —— c's.
21. 13 a's + 4 a's = —— a's.

Copy in columns, and add:

22. 5 bu. 3 pk. 6 qt., 3 bu. 2 pk. 4 qt., 17 bu. 1 pk. 7 qt., 3 bu. 3 pk. 4 qt., 15 bu. 0 pk. 1 qt., 7 bu. 3 pk.

23. 8 T. 560 lb., 7 T. 845 lb., 15 T. 795 lb., 29 T. 1540 lb., 18 T. 1260 lb.

24. 7 bu. 2 pk. 3 qt., 9 bu. 3 pk. 7 qt., 15 bu. 1 pk. 6 qt., 11 bu. 1 pk. 2 qt., 5 bu. 3 pk. 2 qt.

25. 2 T. 1400 lb., 17 T. 1250 lb., 4 T. 1520 lb., 13 T. 560 lb., 28 T. 1750 lb.

26. 7.8, 28.625, 16.5, 29.25, 29.375, 34.25.

27. 1.8, 1.7, 15.5, 28.75, 6.3, 1.04.

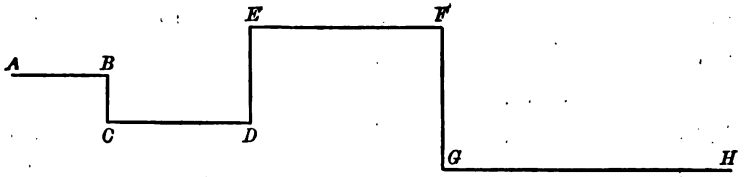
28. 34.15, 20.143, 26.4, 69.35, 27.36, 8.016.

29. 15.5, 37.35, 16.8, 29.62, 2.125, 17.25.

30. $6.85, $5.37, $62.04, $17.09, $8.75, $62.38.

NOTE. — Pupils should aim at accuracy first, then at rapidity.

XIV. ADDITION. SCALE



Scale, $\frac{1}{4}$ inch = 1 mile. AB is $\frac{1}{2}$ inch long.

1. The line AB represents —— miles.
2. The line BC represents —— mile.
3. The line CD represents —— miles.
4. The line DE represents —— miles.
5. The line EF represents —— miles.
6. The line FG represents —— miles.
7. The line GH represents —— miles.
8. How many miles are represented by the broken line $ABCDEFGH$?

9.	10.	11.	12.	13.
$3.94	$4.94	$8.16	$5.06	$0.35
5.50	1.48	8.68	5.74	1.60
11.60	.97	5.36	3.88	4.19
14.84	2.44	1.09	2.36	3.40
5.49	7.12	6.98	1.68	4.01
7.20	6.40	4.06	1.68	33.46
3.68	1.58	6.28	1.49	3.96
6.56	5.07	7.90	1.72	1.52
6.89	5.44	3.22	2.32	5.36
9.96	4.22	3.60	4.05	5.33
5.81	3.89	2.04	1.00	4.70
10.19	7.28	1.68	.58	3.84
8.81	11.92	5.55	.65	3.92
7.36	10.82	4.80	5.08	4.00
3.20	11.64	5.64	1.61	4.24

NOTE. — The last five examples were taken from the pages of a ledger. The pupil should be able to copy and add correctly any one of them in three minutes. It does not require an expert accountant to do this in two minutes.

XV. SUBTRACTION. SIMPLE NUMBERS

1. Subtraction is the process of finding the difference of two numbers.

2. The minuend is the number subtracted from.

3. The subtrahend is the number subtracted.

4. The difference, or remainder, is the answer.

From 683 subtract 249.

683
249
434

9 cannot be taken from 3. Take one unit of the second order from the 8 units. This unit with the 3 units of the first order makes 13 units of the first order. 9 units of the first order from 13 units of the first order leave 4 units of the first order. 4 units of the second order from 7 units of the third order leave 3 units of the second order. 2 units of the third order from 6 units of the third order leave 4 units of the third order. The difference is 434.

Subtract:

5. 5278 1049	**6.** 8394 2187	**7.** 6709 2914	**8.** 7219 2976
9. 9178 2296	**10.** 9012 2193	**11.** 9827 1399	**12.** 7843 1489

13. From the sum of 842, 9684, 8439, and 617, take 14694.

14. A wagon with a load weighed 4970 pounds. The wagon alone weighed 1412 pounds. Find the weight of the load.

15. A horse and rider weighed 1305 pounds. The rider weighed 180 pounds. Find the weight of the horse.

XVI. SUBTRACTION. DECIMALS

From 25.493 subtract 3.5647.

25.493
3.5647
21.9283

Take 1 unit of the third decimal order from the 3 units of that order. It equals 10 units of the fourth decimal order. 7 units of the fourth decimal order from 10 units of that order leave 3 units of the same order. 4 units are greater than 2 units of the same order. Take 1 unit of the second decimal order from the 9 units of

the same order. The 1 unit equals 10 units of the third decimal order. The 10 units and the 2 (3 − 1) units of the third decimal order equal 12 units of that order. 4 units from 12 units leave 8 units. 6 units of the second decimal order from 8 (9 − 1) units of the same order leave 2 units of that order. The 5 units of the first decimal order are greater than the 4 units of the same order. Take 1 primary unit from the 5 primary units. It equals 10 decimal units of the first order. 10 units and 4 units are 14 units. 5 units of the first decimal order from 14 units of the same order leave 9 units of that order. 3 primary units from 4 (5 − 1) primary units leave 1 primary unit. Write the 2 units of the second integral order. The difference is 21.9283.

Subtract:

1. 25.2 − 16.14
2. \$24.30 − 16.25
3. \$86.04 − 23.29
4. 36.19 − 17.284.
5. 7.08 − 1.625.
6. 85.01 − 16.275.
7. 9.076 − 2.63.
8. 47.92 − 18.195.
9. 8 − 3.1793.

XVII. SUBTRACTION. DENOMINATE NUMBERS (*a*)

From 10 bu. 3 pk. 2 qt. subtract 2 bu. 2 pk. 4 qt.

bu.	pk.	qt.
10	3	2
2	2	4
8	0	6

4 qt. being greater than 2 qt., take 1 pk. from the 3 pk. It equals 8 qt. 8 qt. and 2 qt. are 10 qt. 4 qt. from 10 qt. leave 6 qt.

2 pk. from 2 (3 − 1) pk. leave no pecks. 2 bu. from 10 bu. leave 8 bu. The difference is 8 bu. 0 pk. 6 qt.

1. Read the minuend.
2. Read the subtrahend.

3. Read the remainder, or difference.

To test the work, add the difference and subtrahend. The result should equal the minuend.

4. From 16 T. 1825 lb. subtract 7 T. 1240 lb.
5. From 23 T. 1640 lb. subtract 16 T. 1800 lb.
6. From 8 bu. 1 pk. 3 qt. subtract 3 bu. 2 pk. 6 qt.
7. From 16 gal. 2 qt. 1 pt. subtract 4 gal. 3 qt.
8. From 18 gal. 1 qt. subtract 15 gal. 3 qt.
9. From 7 yd. 2 ft. 7 in. subtract 4 yd. 1 ft. 8 in.
10. From 8 yd. 1. ft. subtract 4 yd. 2 ft. 3 in.
11. From 5 yd. 1 ft. subtract 2 yd. 2 ft. 11 in.
12. From 9 yd. subtract 2 yd. 2 ft. 2 in.
13. From 10 A. 24 sq. rd. subtract 3 A. 28 sq. rd.
14. From 24 A. 140 sq. rd. subtract 15 A. 20 sq. rd.
15. From 28 A. 19 sq. rd. subtract 14 A. 100 sq. rd.
16. From 170 A. 116 sq. rd. subtract 138 A. 125 sq. rd.
17. From 248 A. 9 sq. rd. subtract 139 A. 159 sq. rd.

XVIII. SUBTRACTION. DENOMINATE NUMBERS (*b*)

The sign of subtraction ($-$) is read "minus."

Read, filling in the blanks:

1. $12-3=$ ——.
2. $32-3=$ ——.
3. $16-7=$ ——.
4. $26-7=$ ——.
5. $14-5=$ ——.
6. $24-5=$ ——.
7. $17-9=$ ——.
8. $37-9=$ ——.
9. Copy Exs. 1–8, filling in the blanks.

Read, filling in the blanks:

10. 7 6's − 4 6's = —— 6's.
11. 11 4's − 3 4's = —— 4's.
12. 23 7's − 4 7's = —— 7's.
13. 14 5's − 9 5's = —— 5's.
14. 15 8's − 9 8's = —— 8's.
15. 17 3's − 8 3's = —— 3's.
16. 18 x's − 7 x's = —— x's.
17. 17 x's − 8 x's = —— x's.
18. 14 x's − 5 x's = —— x's.
19. 11 b's − 5 b's = —— b's.
20. 15 c's − 7 c's = —— c's.
21. 13 a's − 8 a's = —— a's.

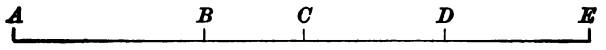
Copy in columns and subtract:

22. 29 bu. 2 pk. 6 qt. − 16 bu. 3 pk. 5 qt.
23. 38 bu. − 16 bu. 3 pk. 5 qt.
24. 24 yd. 2 ft. 8 in. − 17 yd. 1 ft. 9 in.
25. 31 yd. − 16 yd. 2 ft. 8 in.
26. 16 yd. − 4 yd. 1 ft. 11 in.
27. 6 T. 1320 lb. − 4 T. 1640 lb.
28. 25 T. 1120 lb. − 14 T. 1500 lb.

29. 2.9 − .16.
30. 184.5 − 19.6.
31. 46.4 − 17.
32. 84 − 29.75.
33. 125 − 3.6.
34. 84 − 16.6.
35. 186 − 24.7.
36. 67 − 8.34.
37. \$184 − \$2.95.
38. \$63 − \$7.85.
39. \$29 − \$16.65.
40. \$31 − 28.92.
41. 18.25 − 16.375.
42. 56.3 − 15.875.
43. 96.35 − 28.625.
44. 8.45 − 5.925.

NOTE. — Work rapidly and accurately.

XIX. SUBTRACTION. SCALE



Scale, $\frac{1}{4}$ inch to 1 mile.

1. The line AE represents —— miles.
2. The line AB represents —— miles.
3. The line BC represents —— miles.
4. $AB - BC =$ —— miles.
5. $AB - CD =$ —— miles.
6. $AE - DE =$ —— miles.
7. $AE - AB =$ —— miles.
8. $AE - CE =$ —— miles.

6 x's may be written $6x$. 4 a's may be written $4a$. Each is read as written. Read the following, filling in the blanks:

9. $6x - 3x =$ ——.
10. $11x - 5x =$ ——.
11. $15x - 9x =$ ——.
12. $17x - 9x =$ ——.
13. $16x - 7x =$ ——.
14. $13x - 6x =$ ——.
15. $18a - 9a =$ ——.
16. $16a - 5a =$ ——.
17. $14a - 11a =$ ——.
18. $15b - 6b =$ ——.
19. $17b - 9b =$ ——.
20. $15b - 9b =$ ——.
21. $17b - 8b =$ ——.
22. $21c - 2c =$ ——.
23. $21c - 5c =$ ——.
24. $13c - 7c =$ ——.
25. $14c - 6c =$ ——.
26. $15c - 4c =$ ——.

27. From a wagon containing 63 bu. of apples a farmer sold to different customers as follows: 6 bu., $4\frac{1}{2}$ bu., 5 bu., $2\frac{1}{2}$ bu., $10\frac{1}{4}$ bu., $7\frac{1}{2}$ bu., $3\frac{3}{4}$ bu. How many bushels had he left?

28. In 1890 the population of New York was 2,492,591; in 1900 the population was 3,427,202. Find the increase.

XX. REVIEW. ADDITION AND SUBTRACTION

Find what number x stands for in each of these examples:

1. $8 - x = 3$.
2. $34 - x = 15$.
3. $28 - x = 13$.
4. $29 - x = 11$.
5. $x - 21 = 70$.
6. $x - 16 = 40$.
7. $16\,x - 4\,x = 60$.
8. $9\,x - 3\,x = 42$.
9. $28\,x - 20\,x = 72$.
10. $\frac{1}{2}\,x = 10$.
11. $\frac{2}{3}\,x = 12$.
12. $\frac{3}{4}\,x = 24$.

The bank accounts of four depositors were as follows for one day. Find the balance to the credit of each.

13.

JAMES JUDSON

Deposit		$ 624
Check	$ 8.90	
Check	24.75	
Deposit		69.38
Check	38.96	
Check	49.67	
Check	124.28	
Balance ——		

14.

ANNA JENKINS

Deposit		$ 246.80
Deposit		85.75
Check	$ 24.15	
Check	16.45	
Check	85.72	
Check	17.28	
Check	3.75	
Balance ——		

15.

JACOB VAN ORDEN

Deposit		$900
Check	$85.28	
Check	16.94	
Deposit		84
Check	86.49	
Check	75.68	
Check	106.98	
Balance ——		

16.

LOUIS FIELDING

Deposit		$863.42
Check	$96.18	
Check	75.29	
Check	167.43	
Check	19.75	
Deposit		117.46
Check	246.98	
Balance ——		

XXI. MULTIPLICATION

1. *Multiplication* is the process of taking one number as many times as there are units in another.

2. The *multiplicand* is the number which is multiplied, or repeated.

3. The *multiplier* is the number which shows how many times the multiplicand is to be taken.

4. The *product* is the answer.

5. The multiplier is always an abstract number.

6. The product is always of the same denomination as the multiplicand.

7. The sign (×) may be read "multiplied by" or "times." Thus, *$3 × 2* is read *$3 multiplied by 2*; *2 × $3* is read *2 times $3.*

NOTE. — In "Number Foundations" and in "Number Relations," to avoid confusion, the multiplier has always been placed at the right of the sign, and the sign is always to be read *multiplied by;* but from this point on, the pupil must determine from the character of the example which reading should be given.

Read:

8. $\$18 \times 6$.
9. $\$25 \times 8$.
10. $7 \times \$96$.
11. $8 \times \$63$.
12. $15¢ \times 9$.
13. $9 \times 15¢$.
14. $48¢ \times 7$.
15. 24 men $\times$ 5.
16. 5×24 men.
17. 18 bu. $\times$ 9.
18. 9×18 bu.
19. 17 pk. $\times$ 6.
20. 6×17 pk.
21. 7×6. (Either way.)

XXII. MULTIPLICATION. SIMPLE NUMBERS

Multiply 397 by 5.

397 multiplicand.
5 multiplier.
1985 product.

5 times 7 primary units are 35 primary units, or 3 units of the second order and 5 primary units. Write the 5 units. 5 times 9 units of the second order are 45 units of the second order. Adding the 3 units of the same order already obtained gives 48 units of the second order, or 4 units of the third order and 8 units of the second order. Write the 8 units. 5 times 3 units of the third order are 15 units of the third order. Adding the 4 units of this order gives 19 units of the third order. The product is 1985.

Find the product of 397 and 25.

397 multiplicand.
25 multiplier.
1985 } partial products.
794 }
9925 product.

5 times 397 equals 1985. 20 times 397 equals 7940. Twenty times the number and five times the number equal 25 times the number. Hence, $1985 + 7940 = 9925$ is 25 times 397. Note the omission of the cipher in 7940.

Find products:

1. 6943×29.
2. 7296×29.
3. 3694 bu. $\times$ 29.
4. 7542 gal. $\times$ 38.
5. 2574 gal. $\times$ 83.
6. 6789 bbl.* $\times$ 5.
7. 6789 bbl. $\times$ 6.
8. 6789 bbl. $\times$ 7.
9. $8 \times \$384$.†
10. $9 \times \$384$.
11. 25×908 bu.
12. 37×809 bu.
13. 89×708 qt.
14. 98×708 qt.
15. 67×708 gal.
16. 67×806 gal.

XXIII. MULTIPLICATION. DECIMALS

Multiply 13.6 by .7.

$$\begin{array}{r} 13.6 \\ .7 \\ \hline 9.52 \end{array}$$

13.6 multiplied by .7, means, find 7 times one tenth of 13.6. One tenth of 13.6 is 1.36. 7 times 1.36 is 9.52.

How many decimal places in both multiplicand and multiplier? How many in the product?

Multiply 13.6 by .27.

$$\begin{array}{r} 13.6 \\ .27 \\ \hline 1352 \\ 272 \\ \hline 4.072 \end{array}$$

13.6 multiplied by .27, means, find 27 times one hundredth of 13.6. One hundredth of 13.6 is .136. 7 times .136 is 1.352. 20 times .136 is 2.720.

$$1.352 + 2.720 = 4.072.$$

How many decimal places in both multiplicand and multiplier? How many in the product?

* bbl. = barrels.

† See Lesson XXI, 7.

Multiply 3.68 by 21.3.

```
 3.68
 21.3
------
 1104
  368
 736
------
78.384
```

3.68 multiplied by 21.3, means, find 21 times 3.68 plus 3 tenths of 3.68.

.1 of 3.68 = .368.

.3 of 3.68 = 3 times .368	=	1.104
Once 3.68	=	3.68
20 times 3.68	=	73.6
21.3 times 3.68	=	78.384

How many decimal places in both multiplicand and multiplier? How many in the product?

Find products:

1. $897.6 \times .6$.
2. $897.6 \times .36$.
3. 897.6×6.
4. 897.6×36.
5. $967.8 \times .6$.
6. 967.8×3.6.
7. $4.7 \times \$8.23$.
8. $7.4 \times \$8.23$.
9. $4.7 \times \$2.38$.
10. $4.7 \times \$3.82$.
11. $.59 \times 84.2$.
12. 5.9×8.42.

XXIV. MULTIPLICATION. DENOMINATE NUMBERS

Multiply 2 bu. 3 pk. by 5.

```
2 bu.  3 pk.
         5
-------------
13 bu.  3 pk.
```

5 times 3 pk. are 15 pk. 15 pk. = 3 bu. 3 pk. Write the 3 pk. 5 times 2 bu. are 10 bu. 10 bu. and the 3 bu. just obtained equal 13 bu. The product is 13 bu. 3 pk.

1. Find the weight of 6 loads of hay, each weighing 1 T. 780 lb.

2. Find the perimeter of a rectangle 2 yd. 2 ft. long and 1 yd. 2 ft. wide.

3. One side of a certain square is 1 mi. 200 rd. Find its perimeter.

4. A man wishes to build a fence around a rectangular piece of land 20 rd. 4 yd. long and 18 rd. 2 yd. wide. How long a fence will be required?

5. Copy and complete this bill:

NEW YORK, N.Y., July 1, 1904.

MRS. GEORGE LITTELL,

Bought of JONES & Co.

June	3	4 yd. Silk @	$1.50			
		2 Spools Thread @	.08			
	5	8 yd. Cambric @	.07			
		1 Comb			25	
	9	1 Rocker		3	40	
	15	40 yd. Muslin @	.12			
		Received payment,				Amount here
		JONES & Co.				

6. If 2 ten-dollar bills were given by Mrs. Littell in payment of the above bill, how much change should Jones & Co. give back?

XXV. ADDITION, SUBTRACTION, AND MULTIPLICATION

Read:

1. 8 4's $\times 2 =$ —— 4's.
2. 2 9's $\times 5 =$ —— 9's.
3. 4 7's $\times 2 =$ —— 7's.
4. 3×6 5's $=$ —— 5's.
5. 5×7 2's $=$ —— 2's.
6. 9×5 7's $=$
7. 3×9 7's $=$
8. 6×8 4's $=$
9. 7×8 4's $=$
10. 8×9 5's $=$

Copy and complete Exs. 1–10 after reading.

11. $9x \times 3 =$
12. $8x \times 2 =$
13. $12x \times 3 =$
14. $7x \times 5 =$
15. $6x \times 7 =$
16. $4 \times 3x =$
17. $5 \times 8x =$
18. $6 \times 3x =$
19. $9 \times 3x =$
20. $7 \times 3x =$
21. $8 \times 4x =$
22. $5 \times 4x =$
23. $7 \times 4x =$
24. $4 \times 4x =$
25. $9 \times 4x =$

Copy and complete Exs. 11–25 after reading.

26. If x stands for 3 and y stands for 5, then xy stands for 3×5, or 15. $x+y$ stands for the sum of x and y; xy stands for the product of x and y.

Write the sum of a and b.

Write the product of a and b.

27. 7 times x is written $7x$.

Write 8 times y. Write 3 times a times b.

28. If we subtract y from x, we have $x-y$.

Subtract c from b.

29. Write the sum of a and c.

30. Write the difference of a and c.

31. Write the product of a and c.

If $a=3$, $b=4$, and $c=5$, find the number represented by each of these expressions:

32. $2abc$.	**35.** $5a-2b$.	**38.** abc.
33. $a+2b+c$.	**36.** $7a-4c$.	**39.** $5abc$.
34. $3a+2b+c$.	**37.** $6bc-20a$.	**40.** $ab-c$.

XXVI. DIVISION

1. *Division* is the process of finding the number of times that one number is contained in another of the same kind; or, it is the process of finding one of the equal parts of a number.

2. The *dividend* is the number to be divided.

3. The *divisor* is the number by which to divide.

4. The *quotient* is the answer.

5. The sign $\div$ is read "divided by." The dividend is placed at the left of the sign and the divisor at the right. Division is indicated in three ways:

$$18 \div 6 \qquad 6\overline{)18} \qquad \frac{18}{6}$$

If the work is correct, the product of the divisor and quotient should equal the dividend.

NOTE. — It is possible to regard *both* of the following examples as the process of finding how many times one number is contained in another of the same kind. The pupils must be thoughtful and the teacher exceedingly careful if all examples in division be thus regarded — as they *may* be.

(*a*) 4)240 pencils
60 pencils

(*b*) 4 pencils)240 pencils
60 times

In (*a*) we are to find $\frac{1}{4}$ of 240 pencils. We may take 4 pencils from 240 pencils, and put each by itself. Then we may take 4 pencils again, placing 1 pencil with each of the other pencils. Now there are 2 pencils in each place. By continuing this process long enough we shall have all of the 240 pencils in the 4 piles. By counting, we shall find 60 pencils in each pile. One fourth of 240 pencils are as many pencils as 4 pencils are contained times in 240 pencils.

XXVII. DIVISION. SIMPLE NUMBERS

Divide 834 by 6.

6)834
134

One sixth of 8 hundreds is 1 hundred with a remainder of 2 hundreds. 2 hundreds equal 20 tens; 20 tens and 3 tens are 23 tens. One sixth of 23 tens is 3 tens with 5 tens remaining. 5 tens equal 50. 50 and 4 are 54. One sixth of 54 is 9. The quotient is 139. This has been done by "short division."

Divide 7263 by 27.

269
27)7263
54
186
162
243
243

One 27th of 72 hundreds is 2 hundreds with a remainder of 18 hundreds. 18 hundreds equal 180 tens; 180 tens and 6 tens are 186 tens. One 27th of 186 tens is 6 tens with a remainder of 24 tens. 24 tens equal 240; 240 and 3 are 243. One 27th of 243 is 9. The quotient is 269. This has been done by "long division." Long division is usually used when the divisor is more than 12. In "long division" each figure of the quotient is placed directly over the last figure of the partial dividend used.

1. 17,516 ÷ 29.
2. 23,461 ÷ 29.
3. 26,752 ÷ 38.
4. 24,206 ÷ 38.
5. 28,999 ÷ 47.
6. 28,529 ÷ 47.
7. 35,767 ÷ 47.
8. 7849 ÷ 47.
9. 2771 ÷ 163.
10. 8995 ÷ 257.
11. 14,832 ÷ 309.
12. 9158 ÷ 482.
13. 7938 ÷ 567.
14. 69,854 ÷ 659.
15. 162,081 ÷ 783.
16. 241,164 ÷ 783.

XXVIII. DIVISION. DECIMALS

Divide 27.68 by .8.

$$.8\overset{\vee}{)}27.68$$
$$34.6$$

The divisor is tenths, hence place a check mark at the right of tenths in the dividend. Divide as in whole numbers and when the check mark is reached write the decimal point.

8 tenths are contained in 276 tenths 34 times with a remainder of 4 tenths. 4 tenths equal 40 hundredths. 40 hundredths and 8 hundredths are 48 hundredths. 8 tenths are contained in 48 hundredths 6 tenths of a time. The quotient is 34.6.

1. 760 ÷ .8. .8)760.0.*
2. 7.308 ÷ .21. .21)7.308.
3. 180.9 ÷ 22.5. 22.5)180.900.
4. 72 ÷ .25.
5. 725 ÷ 1.25.
6. 48 ÷ .06.
7. 444.5 ÷ 3.5.
8. 266.76 ÷ 7.02.
9. .64 ÷ .008.
10. 64.4 ÷ .7.
11. 84 ÷ .25.
12. 125 ÷ .5

*8 tenths are contained in 7600 tenths —— times.

13. $.049 \div .7$.
14. $.014 \div .0007$. $.0007) .0140$.
15. $53{,}375 \div 12.5$.
16. $53{,}375 \div 1.25$.
17. $53{,}375 \div 1.25$.
18. $53{,}375 \div .0125$.
19. $5337.5 \div 12.5$.
20. $533.75 \div 12.5$.
21. $53.375 \div 12.5$.
22. $31.5 \div 2.25$.
23. $315 \div 2.25$.
24. $3.15 \div 2.25$.
25. $.315 \div 2.25$.
26. $.0315 \div 2.25$.
27. $31.5 \div .225$.
28. $3.15 \div .225$.
29. $33.75 \div 2.25$.
30. $36 \div 2.25$.
31. $38.25 \div 2.25$.
32. $34.125 \div 3.75$.
33. $337.5 \div 3.75$.
34. $300 \div 3.75$.
35. $296.25 \div 3.75$.
36. $292.5 \div 3.75$.
37. $28.875 \div 3.75$.
38. $28.5 \div 3.75$.

XXIX. DIVISION. DENOMINATE NUMBERS

Divide 6 gal. 3 qt. 1 pt. by 4.

$4)\overline{6 \text{ gal. } 3 \text{ qt. } 1 \text{ pt.}}$

$1 \text{ gal. } 2 \text{ qt. } 1\frac{3}{4} \text{ pt.}$

One 4th of 6 gal. is 1 gal. with a remainder of 2 gal. 2 gal. equal 8 qt.; 8 qt. and 3 qt. are 11 qt. One 4th of 11 qt. is 2 qt. with a remainder of 3 qt. 3 qt. equal 6 pt.; 6 pt. and 1 pt. are 7 pt. One 4th of 7 pt. is $1\frac{3}{4}$ pt. The quotient is 1 gal. 2 qt. $1\frac{3}{4}$ pt.

1. The perimeter of a triangle whose sides are equal is 11 rd. 2 yd. 1 ft. 9 in. Find the length of one side.

2. The perimeter of a square is 22 rd. 4 yd. 2 ft. 4 in. Find the length of one side.

3. The perimeter of a regular pentagon (five equal sides) is 12 rd. 3 yd. 0 ft. 6 in. Find the length of one side.

4. 12 5's $\div 3$ 5's $=$
5. 20 4's $\div 2$ 4's $=$
6. 18 9's $\div 3$ 9's $=$
7. 40 8's $\div 5$ 8's $=$
8. 63 7's $\div 9$ 7's $=$
9. 28 6's $\div 4$ 6's $=$
10. 24 7's $\div 3$ 7's $=$
11. 32 9's $\div 4$ 9's $=$
12. 48 3's $\div 6$ 3's $=$
13. 72 2's $\div 8$ 2's $=$

14. $15\,x \div 3\,x =$
15. $18\,x \div 2\,x =$
16. $24\,x \div 6\,x =$
17. $36\,b \div 12\,b =$
18. $60\,b \div 10\,b =$
19. $6\,x \div 6 =$ *
20. $6\,x \div x =$
21. $5\,b \div 5 =$
22. $5\,b \div b =$
23. $3\,a \div 3 =$

24. $96\,a \div 16\,a =$
25. $324\,a \div 18\,a =$
26. $289\,a \div 17\,a =$
27. $256\,b \div 16\,b =$
28. $225\,c \div 15\,c =$
29. $196\,c \div 14\,c =$
30. $169\,d \div 13\,d =$
31. $361\,d \div 19\,d =$

XXX. TESTING RESULTS (*a*)

Casting out nines.— Cast the 9's out of 34,687,943. $3 + 4 + 6 = 13$; subtract 9. $4 + 8 = 12$; subtract 9. $3 + 7 = 10$; subtract 9. $1 + 9 = 10$; subtract 9. $1 + 4 + 3 = 8$. The 8 is less than 9, and it is called the excess of 34,687,943.

* $6\,x$ is the product of 6 and x. Hence, $6\,x \div 6$ must equal x, and $6\,x \div x$ must equal 6. Suppose x to stand for 3. $6\,x$ would stand for 18. $18 \div 6 = 3(x)$. $18 \div 3(x) = 6$.

1. *To check addition by casting out nines,* find the excess of each addend and of the sum. Find the excess of the excesses of the addends. The last excess should equal the excess of the sum.

	Excesses	
9436	4	
7854	6	To find the excess of the excesses of the addends: 4+6=10; subtract 9. Remainder 1+3+4+1=9; subtract 9. Remainder 0+0+5+8=13; subtract 9. Remainder 4+7=11; subtract 9. Remainder 2 = the excess of the excesses.
6789	3	
5674	4	
5896	1	
6849	0	
5936	5	
7469	8	
5947	7	
61850	2	excess of excesses.

2 excess of the sum.

2. Another check for addition is, to add the columns in the opposite direction from that in which they were first added.

3. Another check is, to regard the addends as divided into two or more groups, find the sum of each group, and then the sum of these sums. The sum of the sums should equal the original answer.

Note. — Where more than one method of checking results is possible in any case, let the pupil practice the one that he prefers.

The excess of nines in any number is the remainder found by dividing the number by 9. This always equals the remainder found by dividing the sum of the digits by 9.

XXXI. TESTING RESULTS (*b*)

673	
964	
705	
678	3020
849	
679	
517	
896	2941
5961	5961

This addition is checked by the 3d method. The sum of the first four numbers is 3020. The sum of the remaining four numbers is 2941. By adding these two sums, 5961 is obtained, which agrees with the original answer.

To check subtraction, add the remainder and the subtrahend. The result should equal the minuend.

98463	minuend.
54978	subtrahend.
43485	remainder.

By adding the remainder, 43485, and the subtrahend, 54978, the minuend, 98463, is obtained.

To check multiplication.

849673	1	excess of multiplicand.
8397	0	excess of multiplier.
5947711	0	excess of the product of the excesses.
7647057		
2549019		
6797384		
7134704181	0	excess of the product.

1. *By casting out nines.*—Cast the nines out of the multiplicand. Cast the nines out of the multiplier. Multiply the two excesses together. Find the excess of this product. This excess should equal the excess of the answer.

XXXII. TESTING RESULTS (*c*)

To check multiplication.

1. Divide the product by either the multiplier or the multiplicand, and the quotient should be the other factor. (The pupil should know that the multiplier and the multiplicand are factors of the product.)

To check division.

2. Find the product of the divisor and quotient, and add the remainder. The result should equal the dividend.

Strictly speaking, there is no remainder, if the division is complete; for the remainder is the undivided portion of the dividend.

3. *By casting out nines.*— Find the excess of the divisor and the excess of the quotient. Multiply these two excesses together, and find the excess of their product. Add this last excess and the excess of the remainder. The excess of the result should equal the excess of the dividend.

```
      526
128)67450   128   2  excess of the divisor.
  640       526   4  excess of the quotient.
  345             8  excess of the product of the excesses.
  256       122   5  excess of the remainder.
  890            13  4 excess of the sum of 5 and 8.
  768         67450  4 excess of the dividend.
  122
```

XXXIII. SOME SHORT METHODS. MULTIPLICATION

1. 10 times 8 are ——. 10 times 12 are ——. 10 times 135 are ——. 10 times 45 are ——.

2. 10 times 3.6 are 36. 10 times 5.9 are ——. 10 times 47.35 are 473.5. 10 times 6.84 are ——.

Write the answers of the following:

3.	$10 \times 89 =$	$10 \times 472 =$	$10 \times 396 =$
4.	$10 \times 4.7 =$	$10 \times 5.6 =$	$10 \times 8.5 =$
5.	$10 \times 9.4 =$	$10 \times 8.7 =$	$10 \times 5.2 =$
6.	$10 \times 9.68 =$	$10 \times .869 =$	$10 \times .08 =$
7.	$10 \times .57 =$	$10 \times .057 =$	$10 \times .0057 =$

Notice that the decimal point is moved one place to the right.

8. 100 times 8 are ——. 100 times 12 are ——. 100 times 135 are ——. 100 times 45 are ——.

9. 100 times 3.6 are 360. 100 times 5.9 are ——. 100 times 47.35 are 4735. 100 times 6.84 are ——.

Write the answers of the following:

10.	$100 \times 89 =$	$100 \times 472 =$	$100 \times 396 =$
11.	$100 \times 4.7 =$	$100 \times 5.6 =$	$100 \times 8.5 =$
12.	$100 \times 9.4 =$	$100 \times 8.7 =$	$100 \times 5.2 =$
13.	$100 \times 9.68 =$	$100 \times .869 =$	$100 \times .08 =$
14.	$100 \times .57 =$	$100 \times .057 =$	$100 \times .0057 =$

Notice that the decimal point is moved two places to the right.

To multiply a whole number or a decimal by 10, 100, 1000, *etc., move the decimal point in the multiplicand as many places to the right as there are ciphers in the multiplier, annexing ciphers when necessary.*

Write answers:

15.	$10 \times 6.84 =$	$100 \times 6.84 =$	$1000 \times 6.84 =$
16.	$10 \times .75 =$	$100 \times .75 =$	$1000 \times .75 =$
17.	$10 \times .06 =$	$100 \times .06 =$	$1000 \times .06 =$
18.	$10 \times .0375 =$	$100 \times .0375 =$	$1000 \times .0375 =$

XXXIV. TO MULTIPLY BY 25, $33\frac{1}{3}$, $12\frac{1}{2}$, ETC.

1. $25 =$ —— —— of 100. $33\frac{1}{3} =$ —— —— of 100.

2. $12\frac{1}{2} =$ —— —— of 100. $125 =$ —— —— of 1000.

3. $16\frac{2}{3} =$ —— —— of 100. $250 =$ —— —— of 1000.

4. To multiply a number by 25, multiply it by 100, and divide the product by ——.

5. To multiply a number by $33\frac{1}{3}$, multiply it by 100, and divide the product by ——.

6. To multiply a number by 125, multiply it by 1000, and divide the product by ——.

7. To multiply a number by $16\frac{2}{3}$, multiply it by 100, and divide the product by ——.

8. To multiply a number by 250, multiply it by 1000, and divide the product by ——.

9. Multiply 465 by 25. $465 \times 25 = 11625$.

Think of the 465 as having two ciphers annexed, and then divide 46,500 by 4 without using the pencil, indicating the operation as shown.

10. Multiply 264 by 125. $264 \times 125 = 33000$.

Think of 264 as having three ciphers annexed, and then divide by 8, indicating the work as shown.

11. Multiply 8.496 by $12\frac{1}{2}$. $8.496 \times 12\frac{1}{2} = 106.2$.

Think of 8.496 as having the decimal point moved two places to the right, then divide by 8, indicating the work as shown.

Copy and write the answers:

12.	$48 \times 25 =$	$4.8 \times 25 =$	$.048 \times 25 =$
13.	$642 \times 25 =$	$64.2 \times 25 =$	$.642 \times 25 =$
14.	$96 \times 33\frac{1}{3} =$	$9.6 \times 33\frac{1}{3} =$	$.96 \times 33\frac{1}{3} =$
15.	$374 \times 12\frac{1}{2} =$	$3.74 \times 12\frac{1}{2} =$	$.374 \times 12\frac{1}{2} =$
16.	$685 \times 125 =$	$6.85 \times 125 =$	$.685 \times 125 =$
17.	$84 \times 16\frac{2}{3} =$	$8.4 \times 16\frac{2}{3} =$	$.84 \times 16\frac{2}{3} =$
18.	$176 \times 16\frac{2}{3} =$	$1.76 \times 16\frac{2}{3} =$	$.176 \times 16\frac{2}{3} =$
19.	$931 \times 250 =$	$93.1 \times 250 =$	$.931 \times 250 =$

XXXV. TO DIVIDE BY 10, 100, 1000, ETC.

1.	$24 \div 10 =$	$2.4 \div 10 =$	$.24 \div 10 =$
2.	$245 \div 100 =$	$24.5 \div 100 =$	$2.45 \div 100 =$

To divide a whole number or a decimal by 10, 100, 1000, *etc., move the decimal point as many places to the left as there are ciphers in the divisor.*

Write answers:

3.	$784 \div 10 =$	$78.4 \div 10 =$	$.784 \div 10 =$
4.	$963 \div 100 =$	$96.3 \div 100 =$	$.963 \div 100 =$
5.	$8754 \div 1000 =$	$78.54 \div 1000 =$	$.785 \div 1000 =$
6.	$846 \div 1000 =$	$84.6 \div 1000 =$	$8.46 \div 1000 =$

7. To divide a number by 25, divide it by 100, and multiply the quotient by ——.

8. To divide a number by $33\frac{1}{3}$, divide it by 100, and multiply the quotient by ——.

9. To divide a number by 125, divide it by 1000, and multiply the quotient by ——.

10. To divide a number by $16\frac{2}{3}$, divide it by 100, and multiply the quotient by ——.

11. To divide a number by 250, divide it by 100, and multiply the quotient by ——.

12. Divide 375 by 25. $375 \div 25 = 15$.

Think of 375 as divided by 100, by having the decimal point moved two places to the left, between the 3 and the 7, then multiply the result by 4, indicating the work as shown.

13. Divide 3250 by 125. $3250 \div 125 = 26$.

Think of 3250 as divided by 1000, by having the decimal point moved three places to the left, between the 3 and the 2, then multiply the result by 8, indicating the work as shown.

Copy and write the answers:

14.	$763 \div 25 =$	$58.4 \div 25 =$	$8.46 \div 25 =$
15.	$735 \div 25 =$	$73.5 \div 25 =$	$.735 \div 25 =$
16.	$674 \div 33\frac{1}{3} =$	$67.4 \div 33\frac{1}{3} =$	$.674 \div 33\frac{1}{3} =$
17.	$56 \div 12\frac{1}{2} =$	$5.6 \div 12\frac{1}{2} =$	$.56 \div 12\frac{1}{2} =$
18.	$3125 \div 125 =$	$312.5 \div 125 =$	$.3125 \div 125 =$
19.	$150 \div 16\frac{2}{3} =$	$1.5 \div 16\frac{2}{3} =$	$.15 \div 16\frac{2}{3} =$
20.	$2415 \div 16\frac{2}{3} =$	$24.15 \div 16\frac{2}{3} =$	$.2415 \div 16\frac{2}{3} =$
21.	$750 \div 250 =$	$75 \div 250 =$	$.75 \div 250 =$

XXXVI. TO DIVIDE BY ANY MULTIPLE OF TEN. REVIEW

1. Divide 2843 by 60.

$6|0\overline{)284|3}$ $\quad 47\frac{23}{60}$

Cut off the cipher and the right-hand figure 3, as shown. Dividing the 284 by 6 we obtain 47, with a remainder of 2. This 2 prefixed to the 3 forms 23, which we divide by 60, placing the quotient as part of complete quotient, $47\frac{23}{60}$.

2. Divide 702,130 by 26,000.

$$26|000\overline{)702|130} \quad 27\tfrac{1}{200}$$

$$\begin{array}{r} 52 \\ \hline 182 \\ 182 \\ \hline \end{array}$$

$\frac{130}{26000} = \frac{1}{200}$

Cut off the three ciphers in the divisor, as shown. Then cut off as many places from the right of the dividend. Dividing 702 by 26, we obtain 27. Then dividing the 130 by 26,000, and reducing the resulting fraction to its lowest terms, we obtain $\frac{1}{200}$. The quotient is $27\frac{1}{200}$.

3. Divide 846 by 50.
4. Divide 7496 by 70.
5. Divide 67,840 by 800.
6. Divide 6472 by 60.
7. Divide 1540 by 700.
8. Divide 12,150 by 2430.
9. Divide 602,430 by 86,000.

Review:

10. $840 \times 16\frac{2}{3} =$
11. $764 \div 16\frac{2}{3} =$
12. $960 \times 25 =$
13. $960 \div 25 =$
14. $129 \times 33\frac{1}{3} =$
15. $129 \div 33\frac{1}{3} =$
16. $960 \times 12\frac{1}{2} =$
17. $960 \div 12\frac{1}{2} =$
18. $84 \div 100 =$
19. $8.4 \times 100 =$
20. $2.34 \times 1000 =$
21. $2.34 \div 1000 =$

XXXVII. CANCELLATION

1. $64 \div 16 =$ ——. If we divide both numbers by 4, we obtain 16 and 4. $16 \div 4 =$ ——. If we divide both numbers by 2, the quotient remains the same. Dividing both dividend and divisor by the same number does not affect the quotient. Therefore, dividing both the numerator and the denominator of a fraction by the same number does not change its value.

Dividing the dividend and the divisor by the same number is called cancellation, because that number is thus canceled or rejected from the two numbers.

2. Divide 6×24 by 2×4.

$$\frac{\overset{3}{\cancel{6}} \times \overset{6}{\cancel{24}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{4}}} = \frac{18}{1} = 18.$$

2 is a common factor of 6 and 2. Dividing 6 and 2 by it, we obtain 3 and 1. Dividing 24 and 4 by their common factor, 4, we obtain 6 and 1. The product of 3 and 6 is 18. The product of 1 and 1 is 1. $\frac{18}{1} = 18$. It is evident that the 1's in the divisor need not be written.

3. Divide $6 \times 36 \times 85$ by $3 \times 12 \times 17$.

4. Divide $24 \times 3 \times 51$ by 36×17.

5. Divide $45 \times 18 \times 35$ by $5 \times 7 \times 27$.

6. Divide $4 \times 84 \times 4$ by $4 \times 8 \times 4$.

7. Divide $24 \times 6 \times 1728$ by $4 \times 12 \times 60$.

8. Divide $65 \times 75 \times 100$ by $25 \times 13 \times 70$.

XXXVIII. MISCELLANEOUS EXERCISES

1. A farmer bought 230 sheep for \$4.40 a head. He sold 54 of them for \$5 a head, 67 for \$4.90 a head, and the rest for \$5.25 a head. How much was his profit?

2. Mr. Abrams sold 6 cords of wood for \$4.50 a cord, and took in payment sheep at \$3 a head. How many sheep did he get? (Cancellation.)

3. Write in the Arabic System, three million eighty-nine thousand fourteen.

4. Write in the Roman System, 1904.

5. How much will a pile of wood 42 ft. long, 4 ft. wide, and 8 ft. high cost at \$5 a cord? A cord is 128 cu. ft. (Cancellation.)

6. At 75¢ a bushel, how many bushels of wheat can be bought for \$240?

7. At 35¢ a bushel, how many bushels of potatoes can be bought for \$133?

8. At \$1.05 a bushel, how many bushels of wheat can be bought for \$215.25?

9. A grocer bought 14 dozen eggs for 16 cents a dozen. Six eggs were broken. He sold the rest at 18 cents a dozen. What was his profit?

10. A grocer bought 23 dozen eggs at 18 cents a dozen. Eight eggs were broken. He sold the rest at 24 cents a dozen. How much was his gain?

11. A man fed each of his 6 cows 3 quarts of wheat bran twice a day. In how many days will he feed $31\frac{1}{2}$ bushels? 32 qt. = 1 bu.

12. Find the cost of a rectangular piece of land 80 rd. long and 60 rd. wide at $85 per acre.

13. Find the cost of 612 pounds of meat at $12\frac{1}{2}$ cents a pound.

14. 231 cubic inches are one gallon. How many gallons in 70,455 cu. in.?

15. How many cords in 26,752 cu. ft.?

16. Find the cost of 10,968 eggs at 23¢ a dozen.

17. Find the wages earned by a workman in 216 hours at $1.75 a day, of 8 hours each.

18. How many bushels in 28,992 quarts?

19. Find the cost of 50,400 lb. of wheat at 85 cents a bushel. A bushel of wheat weighs 60 lb.

20. At $2.25 a barrel, how many barrels of potatoes can be bought for $94.50?

21. If 8 calves cost $92, what will 16 calves cost at the same rate?

22. If $5x$ equals 960, what does x equal?

23. $5 \times 2x = 100$. Find the value of x.

24. $8x + 3x = 99$. Find the value of x.

25. Find the cost of fencing a rectangular piece of ground 90 rd. by 60 rd., at 94 cents per rod.

26. Find the value of a rectangular piece of ground 90 rd. by 60 rd., at \$108 per acre.

27. An acre contains 43,560 square feet. How many acres in a rectangular plot 3960 ft. long and 1067 ft. wide?

28. $126 \div .6$.

29. $70.6 \div .04$.

30. 13.8×3.5.

31. 7.04×1.5.

Write the answers:

32. $88 \times 12\frac{1}{2}$.

33. 84×25.

34. $69 \times 33\frac{1}{3}$.

35. $906 \times 16\frac{2}{3}$.

36. $24 \div 12\frac{1}{2}$.

37. $750 \div 25$.

38. $200 \div 33\frac{1}{3}$.

39. $150 \div 16\frac{2}{3}$.

XXXIX. LEAST COMMON MULTIPLE

1. A common multiple of 3 and 4 is ——, or ——, or ——.

2. The least common multiple of 3 and 4 is ——.

3. A common multiple of two or more numbers is any number which is exactly divisible by them.

4. The least common multiple of two or more numbers is the least number which is exactly divisible by them.

5. Find the least common multiple of 42, 75, and 63.

$$\begin{array}{r|l} 2 & 42 \\ 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 75 \\ 5 & 25 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ 3 & 21 \\ \hline & 7 \end{array}$$

$42 = 2 \times 3 \times 7$

$75 = 3 \times 5 \times 5$

$63 = 3 \times 3 \times 7$

L. C. M. $= 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 3150$.

A number to be exactly divisible by 42 must have the prime factors of 42, which are 2, 3, and 7.

A number to be exactly divisible by 75 must have the prime factors of 75, which are 3, 5, and 5.

A number to be exactly divisible by 63 must have the prime factors of 63, which are 3, 3, and 7.

It is then evident that no factor need be taken more than once, unless it occurs more than once in some one number.

The least common multiple of several numbers is never —— than the largest number.

We may easily find the least common multiple by inspection, when the numbers are small.

6. Find the least common multiple of 8, 3, 12, 9.

The L. C. M. cannot be less than 12. The next multiple of 12 is 2 times 12, or 24. 24 is a multiple of 8 and 3, but not of 9. Then try 3 times 12, or 36. 36 is divisible by 8, 3, and 9, and it is therefore the L. C. M. required.

Find the least common multiple of:

7. 8, 9, 24, 18.

8. 91, 42, 65, 26.

9. 52, 104, 117, 130.

10. 2, 3, 4, 6, 7.

11. 8, 56, 84, 28.

12. 30, 35, 70, 25.

13. 18, 21, 24, 15.

14. 30, 40, 50, 60.

XL. LEAST COMMON MULTIPLE. SECOND METHOD

1. Find the least common multiple of 42, 75, and 63.

3	42	75	63
7	14	25	21
	2	25	3

Since 3 is a prime factor of more than one of the numbers, we divide by 3, writing the quotients in a line below. Since 7 is a prime factor of more than one number in the second row, we divide by 7, writing the quotients and undivided number as before. No two of these numbers have a common prime factor.

The L. C. M. is $3 \times 7 \times 2 \times 25 \times 3 = 3150$.

Find the least common multiple of:

2. 10, 25, 42, 60.
3. 12, 18, 6, 42.
4. 45, 60, 25, 35.
5. 24, 36, 48, 72.
6. 15, 18, 21, 24.
7. 12, 16, 20, 24.
8. 2, 3, 4, 5, 6, 7, 8.
9. 91, 63, 42, 84.
10. 104, 65, 26, 39.
11. 22, 33, 44, 55.
12. 17, 34, 51, 68.
13. 52, 65, 26, 13.

8 and 15, although neither is a prime number, are said to be prime to each other because they have no common prime factor. Their least common multiple is their product.

The least common multiple of numbers prime to each other is their product.

The least common multiple of several numbers is the product of all their different prime factors.

By comparing the two methods, it will be observed that only the different prime factors are selected in the first case, and only the different prime factors are found in the second case.

XLI. FRACTIONS. REDUCTION

Change $\frac{16}{24}$ to lowest terms.

$\frac{16}{24} = \frac{2}{3}$. Divide both terms by 8.

Change to lowest terms:

1. $\frac{18}{72}$.
2. $\frac{24}{120}$.
3. $\frac{63}{180}$.
4. $\frac{25}{75}$.
5. $\frac{17}{34}$.
6. $\frac{19}{57}$.
7. $\frac{15}{95}$.
8. $\frac{39}{52}$.
9. $\frac{84}{124}$.
10. $\frac{21}{84}$.
11. $\frac{34}{85}$.
12. $\frac{26}{65}$.

Change $8\frac{2}{3}$ to an improper fraction.

$8\frac{2}{3} = \frac{26}{3}$. One whole is 3 thirds. 8 wholes = 24 thirds. $\frac{24}{3} + \frac{2}{3} = \frac{26}{3}$.

Change to improper fractions:

13. $7\frac{2}{5}$.
14. $64\frac{5}{6}$.
15. $29\frac{2}{3}$.
16. $17\frac{5}{6}$.
17. $83\frac{2}{7}$.
18. $903\frac{1}{3}$.
19. $87\frac{3}{4}$.
20. $92\frac{1}{5}$.
21. $106\frac{2}{3}$.
22. $94\frac{7}{9}$.
23. $601\frac{3}{5}$.
24. $89\frac{4}{7}$.

Change $\frac{84}{5}$ to a mixed number.

$\frac{84}{5} = 16\frac{4}{5}$. In 5 fifths there is 1 whole. In 84 fifths there are as many wholes as 5 fifths are contained times in 84 fifths.

Change to whole or mixed numbers:

25. $\frac{372}{6}$.
26. $\frac{804}{2}$.
27. $\frac{974}{13}$.
28. $\frac{806}{15}$.
29. $\frac{715}{16}$.
30. $\frac{804}{15}$.

XLII. ADDITION AND SUBTRACTION

Add 1 fourth and 1 third.

1 fourth is 3 twelfths. 1 third is 4 twelfths. 3 twelfths and 4 twelfths are 7 twelfths.

Add $\frac{3}{4}$ and $\frac{2}{3}$.

3 fourths are 9 twelfths. 2 thirds are 8 twelfths. 9 twelfths and 8 twelfths are 17 twelfths.

Before fractions can be added, they must have the same name, that is, a common denominator.

The most convenient common denominator is the least common multiple of the denominators.

Add $\frac{5}{6}$ and $\frac{3}{4}$. The L. C. M. of the denominators, 6 and 4, is 12. Change the fractions to 12ths. $\frac{1}{6}$ equals —— twelfths. $\frac{5}{6}$ equals —— twelfths. $\frac{1}{4}$ equals —— twelfths. $\frac{3}{4}$ equals —— twelfths. —— twelfths and —— twelfths are —— twelfths. $\frac{19}{12} = 1\frac{7}{12}$.

Add $16\frac{1}{2}$, $24\frac{2}{3}$, and $106\frac{3}{4}$.

	12
$16\frac{1}{2}$	6
$24\frac{2}{3}$	8
$106\frac{3}{4}$	9
$147\frac{11}{12}$	$\frac{23}{12} = 1\frac{11}{12}$

Write the addends as shown. The L. C. M. of 2, 3, and 4 is 12. Write the 12 as shown. $\frac{1}{2}$ equals 6 twelfths. $\frac{2}{3}$ equals 8 twelfths. $\frac{3}{4}$ equals 9 twelfths. The numbers in the column headed by 12 are all 12ths. Adding, we obtain $\frac{23}{12}$. $\frac{23}{12} = 1\frac{11}{12}$. Write $\frac{11}{12}$ under the column of fractions. Add the 1 to the numbers in the right-hand column.

Add $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{3}$, and $\frac{5}{8}$.

$$\frac{3}{4} + \frac{5}{6} + \frac{2}{3} + \frac{5}{8} = \frac{18 + 20 + 16 + 15}{24} = \frac{69}{24} = 2\frac{7}{8}.$$

Or, the fractions may be arranged in a column as in the preceding exercise.

From $\frac{7}{8}$ subtract $\frac{2}{3}$. $\quad \frac{7}{8}-\frac{2}{3}=\frac{21-16}{24}=\frac{5}{24}$.

Or,

	24	
$\frac{7}{8}$	21	The L. C. M. of 8 and 3 is 24. $\frac{7}{8}=\frac{21}{24}$. $\frac{2}{3}=\frac{16}{24}$.
$\frac{2}{3}$	16	$\frac{21}{24}-\frac{16}{24}=\frac{5}{24}$.
$\frac{5}{24}$	5	

From 400 take $8\frac{5}{9}$.

400	$400=399\frac{9}{9}$. $\frac{5}{9}$ from $\frac{9}{9}=\frac{4}{9}$. 8 from 399 = 391. $\therefore 400-8\frac{5}{9}$
$8\frac{5}{9}$	$=391\frac{4}{9}$.
$391\frac{4}{9}$	

From $305\frac{2}{3}$ take $29\frac{7}{8}$.

	24	
$305\frac{2}{3}$	16	(40)
$29\frac{7}{8}$	21	
$275\frac{19}{24}$	19	

Write the minuend and subtrahend as shown. The L. C. M. of 3 and 8 is 24. Change the fractions to 24ths. $\frac{2}{3}=\frac{16}{24}$. $\frac{7}{8}=\frac{21}{24}$.

As 21 is larger than 16, take 1 from 5, and, calling it 24 twenty-fourths, add it to 16 twenty-fourths, obtaining 40 twenty-fourths. 40 twenty-fourths less 21 twenty-fourths equal 19 twenty-fourths. Then subtract 29 from 304. The remainder is $275\frac{19}{24}$.

1. $\frac{2}{3}+\frac{3}{5}$.
2. $\frac{1}{2}+\frac{7}{8}$.
3. $\frac{2}{3}+\frac{1}{2}$.
4. $\frac{2}{3}-\frac{3}{5}$.
5. $\frac{7}{8}-\frac{1}{2}$.
6. $\frac{2}{3}-\frac{1}{2}$.
7. $8\frac{1}{2}+3\frac{2}{3}$.
8. $6\frac{5}{6}+4\frac{3}{4}$.
9. $9\frac{1}{4}+3\frac{1}{5}$.
10. $8\frac{2}{3}-3\frac{1}{2}$.
11. $28\frac{1}{2}+26\frac{5}{6}+27\frac{7}{9}$.
12. $1842\frac{3}{5}+1903\frac{5}{6}$.
13. $1903\frac{1}{6}-1842\frac{3}{5}$.
14. $9\frac{5}{6}+3\frac{1}{5}+6\frac{2}{3}$.
15. $846\frac{2}{5}-197\frac{1}{2}$.
16. $17\frac{3}{8}+19\frac{5}{6}+8\frac{7}{12}$.
17. $271\frac{3}{8}-104\frac{3}{4}$.
18. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$.
19. $\frac{2}{5}+\frac{5}{6}+\frac{3}{10}+\frac{1}{2}$.
20. $\frac{7}{15}+\frac{2}{3}+\frac{1}{2}+\frac{9}{10}$.

XLIII. REDUCTION

Change $\frac{3}{8}$ to a decimal.

$\frac{3}{8}$ may mean $\frac{1}{8}$ of 3. $8\overline{)3.000}$ $\therefore$ $\frac{3}{8} = .375$.
$.375$

To change a common fraction to a decimal, *annex decimal ciphers to the numerator and divide by the denominator.*

Change .375 to a common fraction.

$.375 = \frac{375}{1000} = \frac{75}{200} = \frac{3}{8}$.

To change a decimal fraction to a common fraction, *write the numerator and the denominator of the decimal in the form of a common fraction, and reduce the fraction to lowest terms.* Omit the decimal point.

The denominator of a decimal may be expressed by 1 with as many ciphers annexed as there are decimal places in the decimal.

Change to decimals:

1. $\frac{3}{5}$.
2. $\frac{3}{4}$.
3. $\frac{7}{8}$.
4. $\frac{3}{8}$.
5. $\frac{3}{16}$.
6. $\frac{5}{16}$.
7. $\frac{4}{5}$.
8. $\frac{9}{16}$.
9. $\frac{7}{25}$.
10. $4\frac{7}{8}$.
11. $5\frac{1}{16}$.
12. $9\frac{5}{32}$.
13. $4\frac{29}{64}$.
14. $8\frac{1}{8}$.
15. $91\frac{3}{25}$.
16. $72\frac{31}{32}$.
17. $20\frac{4}{25}$.
18. $19\frac{21}{25}$.
19. $7\frac{3}{16}$.
20. $9\frac{3}{20}$.

Change to common fractions:

21. .125.
22. .375.
23. .871½.
24. .625.
25. .1875.
26. .3125.
27. .28.
28. .5625.
29. .21875.
30. .025.
31. .002.
32. .0125.

XLIV. MULTIPLICATION — COMMON FRACTIONS

(1) Find the cost of 2 bu. of apples at $\$\frac{3}{4}$ a bushel.

2 bu. cost $\quad \frac{2}{1} \times \$\frac{3}{4} = \$\frac{6}{4} = \$1\frac{1}{2}.$

(Cancellation) $\quad \overset{1}{\cancel{2}} \times \$\frac{3}{\cancel{4}} = \$\frac{3}{2} = \$1\frac{1}{2}.$
$\qquad\qquad\qquad\qquad\qquad\quad 2$

(2) Find the cost of $\frac{3}{4}$ yd. of cloth at $\$6$ a yard.

$\frac{3}{4}$ yd. costs $\quad \frac{3}{4}$ of $\$\frac{6}{1} = \$\frac{18}{4} = \$4\frac{1}{2}.$

(Cancellation) $\quad \frac{3}{\cancel{4}}$ of $\$\overset{3}{\cancel{6}} = \$\frac{9}{2} = \$4\frac{1}{2}.$
$\qquad\qquad\qquad\qquad\quad 2$

(3) Find the cost of $2\frac{3}{4}$ bu. of potatoes at $\$1\frac{1}{3}$ a bushel.

$2\frac{3}{4}$ bu. cost $\quad \frac{11}{4} \times \$\frac{4}{3} = \$\frac{44}{12} = \$3\frac{2}{3}.$

(Cancellation) $\quad \frac{11}{\cancel{4}} \times \$\frac{4}{3} = \$\frac{11}{3} = \$3\frac{2}{3}.$

(4) Change $\frac{3}{4}$ ft. to inches.

$$\frac{3}{4} \text{ ft.} = \frac{3}{\cancel{4}} \text{ of } \frac{\overset{3}{\cancel{12}}}{1} \text{ in.} = 9 \text{ in.}$$

Find the cost of:

1. 16 bu. of corn at $\$\frac{3}{4}$ a bushel.
2. $\frac{3}{5}$ acre of land at $\$160$ an acre.
3. $4\frac{2}{3}$ barrels of apples at $\$1\frac{1}{2}$ a barrel.
4. $\frac{2}{3}$ yd. of cloth at $\$1\frac{1}{2}$ a yard.

5. 15 acres of land at $\$118\frac{3}{5}$ an acre.

$\$118\frac{3}{5}$
15

9	$(15 \times \frac{3}{5})$
590	(5×118)
118	(10×118)

$\$1779$

When the integral part of the mixed number is large, it is usually better to multiply the integral and fractional parts separately, as shown here, than to change the mixed number to an improper fraction.

6. Thirty pounds of sugar were bought at 4.5¢ and sold for $\$1.50$. At the same rate what would be the gain on 2 barrels of 200 pounds each?

7. 18 acres of land bought at $\$235\frac{2}{5}$ an acre were sold at $\$284\frac{4}{5}$ an acre. Find the gain.

XLV. DIVISION — COMMON FRACTIONS

ORAL

1. If $2\frac{1}{3}$ lb. of cherries cost 28¢, $\frac{1}{3}$ of a pound costs —— cents and 1 lb. costs —— cents. $28 \div 2\frac{1}{3} = \frac{28}{1} \times \frac{3}{7} =$

To divide by a fraction, *invert the divisor and then do as in multiplication.*

2. 5 lb. of coffee cost $\$1.75$. One pound costs ——.

3. $2\frac{1}{3}$ lb. of sugar cost 14¢. One pound costs ——.

4. $1 \div \frac{1}{2} =$ $1 \div \frac{1}{5} =$ $1 \div \frac{1}{10} =$

5. $1 \div \frac{1}{3} =$ $1 \div \frac{1}{6} =$ $1 \div \frac{1}{15} =$

6. $1 \div \frac{1}{4} =$ $1 \div \frac{1}{7} =$ $1 \div \frac{1}{24} =$

7. Since $1 \div \frac{1}{3} =$ ——, $1 \div \frac{2}{3} = \frac{1}{2}$ of ——, or ——.
$1 \div \frac{2}{3} = \frac{3}{2}$ of 1. $(\frac{1}{1} \times \frac{3}{2}.)$

8. Since $2 \div \frac{1}{3} =$ ——, $2 \div \frac{2}{3} = \frac{1}{2}$ of ——, or ——.
$2 \div \frac{2}{3} = \frac{3}{2}$ of 2. $(\frac{2}{1} \times \frac{3}{2}.)$

9. $\frac{3}{5} \div \frac{2}{7} = \frac{21}{35} \div \frac{10}{35} = \frac{21}{10} = 2\frac{1}{10}$. $\frac{3}{5} \div \frac{2}{7} = \frac{3}{5} \times \frac{7}{2} = \frac{21}{10} = 2\frac{1}{10}$.

10. $\frac{5}{9} \div \frac{2}{3} = \frac{5}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{2} = \frac{5}{6}$.

11. $\frac{7}{8} \div \frac{3}{4} = \frac{7}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{3} = \frac{7}{6} = 1\frac{1}{6}$.

WRITTEN

(Cancellation may be used.)

12. $\frac{3}{5} \div \frac{7}{10}$.

13. $\frac{6}{7} \div \frac{4}{9}$.

14. $\frac{3}{4} \div \frac{9}{10}$.

15. $\frac{5}{17} \div \frac{15}{34}$.

16. $\frac{3}{8} \div \frac{5}{16}$.

17. $\frac{5}{32} \div \frac{13}{16}$.

18. $\frac{5}{11} \div \frac{2}{5}$.

19. $\frac{4}{25} \div \frac{16}{35}$.

20. $\frac{25}{64} \div \frac{5}{8}$.

Change mixed numbers to improper fractions:

21. $5\frac{1}{3} \div 2\frac{3}{4}$.

22. $8\frac{4}{5} \div 5\frac{1}{3}$.

23. $7\frac{1}{8} \div \frac{5}{19}$.

24. $5\frac{2}{3} \div \frac{34}{35}$.

25. $6\frac{1}{2} \div \frac{2}{13}$.

26. $\frac{5}{8} \div 2\frac{1}{5}$.

27. $2\frac{1}{3} \div 4\frac{1}{2}$. This may be written $\frac{2\frac{1}{3}}{4\frac{1}{2}}$. It is then called a complex fraction. It may be changed to a simple fraction by multiplying each term by a common multiple of the denominators. $\frac{2\frac{1}{3} \times 6}{4\frac{1}{2} \times 6} = \frac{14}{27}$.

28. $\frac{37\frac{1}{2}}{100} =$

29. $\frac{87\frac{1}{2}}{100} =$

30. $\frac{66\frac{2}{3}}{100} =$

31. $\frac{\frac{5}{8}}{\frac{3}{4}} =$

XLVI. REVIEW

1. Write the ten characters of the Arabic Notation. (L. I : 1.)

2. Write the seven characters of the Roman Notation. (L. I : 2.)

3. Write 1905 by both methods.

4. Write in figures four hundred seven thousandths. (L. VII : 4.)

5. Write in figures eight hundred and six thousandths. (L. VII : 7.)

6. From 28.7 take 9.28. (L. XVI : 1.)

7. From 9 yd. 1 ft. take 2 yd. 2 ft.

8. A woman bought 6 yd. of silk at $2.50 a yard, and 5 yd. of calico at 7¢ a yard, giving in payment 4 five-dollar bills. How much change ought she to receive?

9. What does x stand for in the expression, $x - 94 = 189$?

10. What does x stand for in the expression, $2x + 4 = 96$?

11. What does x stand for in the expression, $\frac{3}{4}x = 84$?

12. Find the cost of 3.75 acres of land at $84.20 an acre.

13. If 74.6 acres of land cost $5401.04, what was the price per acre?

14. How many tons in 12 bales of hay, each weighing 240 lb.?

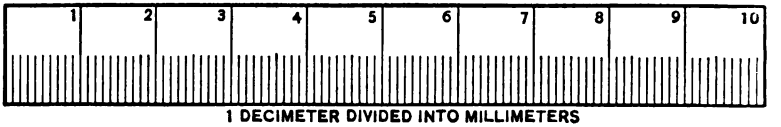
If $x = 8$ and $y = 6$, find the value of: (L. XXV.)

15. $x + y$. 16. $x - y$. 17. xy.

18. $6x + 9y$. 19. $21x - 21y$. 20. $7xy$.

XLVII. THE METRIC SYSTEM

The Metric System of Weights and Measures has been legalized by nearly all civilized nations. It originated in France, and takes its name from the *meter*, the fundamental unit.



1 DECIMETER DIVIDED INTO MILLIMETERS

This line is one decimeter or one tenth of a meter in length. Each division of it is one centimeter or one hundredth of a meter in length. One tenth of the centimeter is one millimeter in length.

Deci means tenth,
centi means hundredth,
milli means thousandth.

Therefore, a decimeter is one tenth of a meter, a centimeter is one hundredth of a meter, and a millimeter is one thousandth of a meter.

A line ten meters long is one dekameter in length.

A line one hundred meters long is one hektometer in length.

A line one thousand meters long is one kilometer in length.

Deka means ten,
hekto means one hundred,
kilo means one thousand.

Therefore, a dekameter is ten meters, a hektometer is one hundred meters, and a kilometer is one thousand meters.

XLVIII. LINEAR MEASURES

10 millimeters (mm) = 1 centimeter (cm).
10 centimeters = 1 decimeter (dm).
10 decimeters = 1 *meter* (m).
10 meters = 1 dekameter (Dm).
10 dekameters = 1 hektometer (Hm).
10 hektometers = 1 kilometer (Km).

The myriameter (10,000 meters) is seldom used.

1. Cut a piece of paper or pasteboard one decimeter long and mark it carefully into centimeters. (See the decimeter line in Lesson 47.)

2. How long is your arithmetic? (Measure with your decimeter.)

3. How wide is your arithmetic?

4. How many decimeters in 2 meters?

5. How many decimeters in 3.7^{m}?

6. $8^{m} = \_\_\_\_\_^{dm} = \_\_\_\_\_^{cm} = \_\_\_\_\_^{mm}$.

7. $7^{Dm} = \_\_\_\_\_^{m} = \_\_\_\_\_^{dm} = \_\_\_\_\_^{cm} = \_\_\_\_\_^{mm}$.

8. $3^{Km} = \_\_\_\_\_^{Hm} = \_\_\_\_\_^{Dm} = \_\_\_\_\_^{m} = \_\_\_\_\_^{dm} = \_\_\_\_\_^{mm}$.

9. $125^{m} = \_\_\_\_\_^{Dm} = \_\_\_\_\_^{Hm} = \_\_\_\_\_^{Km}$.

10. $348^{dm} = \_\_\_\_\_^{m} = \_\_\_\_\_^{Dm} = \_\_\_\_\_^{Hm} = \_\_\_\_\_^{Km}$.

11. $35{,}468^{cm} = \_\_\_\_\_^{m} = \_\_\_\_\_^{Km}$.

12. $84.69^{m} = \_\_\_\_\_^{Km} = \_\_\_\_\_^{mm}$.

13. $93.275^{m} = \_\_\_\_\_^{Hm} = \_\_\_\_\_^{cm}$.

14. $74{,}396^{mm} = \_\_\_\_\_^{m} = \_\_\_\_\_^{Km}$.

Change to meters and add:

15. 453^{cm}, 983^{mm}, 839^{dm} and 8.36^{Dm}.

16. 8^{Km}, 64^{Dm}, 49.85^{Hm}, 385^{dm} and 184^{cm}.

17. 83.74^{cm}, 96^{mm}, 13^{dm} and 48.3^{Dm}.

18. 43^{Dm}, 8437^{mm}, 69^{Dm} and 84^{Km}.

XLIX. SURFACE MEASURES

1. Draw a decimeter square.

2. Mark it off into as many centimeter squares as possible.

3. In one square decimeter there are —— square centimeters.

4. A centimeter square is —— millimeters long and —— millimeters wide. It contains —— square millimeters.

100 square millimeters $^{(qmm)}$ = 1 square centimeter $^{(qcm)}$.
100 square centimeters = 1 square decimeter $^{(qdm)}$.
100 square decimeters = 1 *square meter* $^{(qm)}$.
100 square meters = 1 square dekameter $^{(qDm)}$.
100 square dekameters = 1 square hektometer $^{(qHm)}$.
100 square hektometers = 1 square kilometer $^{(qKm)}$.

In measuring land the square meter, the square dekameter, and the square hektometer are used, but other names are assigned.

The square meter is called the *centare* $^{(ca)}$, the square dekameter is called the *are* $^{a)}$, and the square hektometer is called the *hektare* $^{(Ha)}$.

Land Measures

100 centares $^{(ca)}$ = 1 are $^{(a)}$.

100 ares = 1 hektare $^{(Ha)}$.

Copy and fill the blanks:

5. $3^{qKm} =$ _____ $^{qHm} =$ _____ $^{qDm} =$ _____ ^{qm}.
6. $8.35^{qHm} =$ _____ $^{qDm} =$ _____ $^{qm} =$ _____ ^{qdm}.
7. $7.53^{qDm} =$ _____ $^{qm} =$ _____ $^{qdm} =$ _____ ^{qcm}.
8. $58.47^{qm} =$ _____ $^{qdm} =$ _____ $^{qcm} =$ _____ ^{qmm}.
9. $84^{qm} =$ _____ $^{qDm} =$ _____ $^{qHm} =$ _____ ^{qKm}.
10. $964^{qdm} =$ _____ $^{qm} =$ _____ $^{qDm} =$ _____ ^{qHm}.
11. $547^{qcm} =$ _____ $^{qdm} =$ _____ $^{qm} =$ _____ ^{qDm}.
12. $835^{qmm} =$ _____ $^{qcm} =$ _____ $^{qdm} =$ _____ ^{qm}.

L. PROBLEMS

1. How many centares in 52 ares?

2. How many centares in 52 hektares?

3. How many hektares in a rectangular piece of land 600^{m} long and 400^{m} wide?

4. Find the cost of a rectangular piece of land 500^{m} long and 300^{m} wide, at \$240 per hektare.

5. Change to hektares and add: 284^{ca}, 58.25^{a}, 69.2^{Ha}, 84^{Ha} and 9.87^{a}.

6. From 86.78^{Ha} a man sold 964.75^{ca}. How many hektares did he still own?

7. If a merchant's store is 2.3^{Km} from his home, how many kilometers will he travel in making two round trips to his store?

8. If a man's steps average 8^{dm} in length, how many steps will he take in walking 28^{Km}?

9. If a man's steps average 8^{dm} in length, how many kilometers will he walk in taking 3600 steps?

10. The circumference of a certain wheel (the distance around it) is 6^{m}. How many turns will it make in rolling 6^{Km}?

11. The fore wheel of a wagon is 3^{m} in circumference and the hind wheel is 3.5^{m} in circumference. How many more turns will the fore wheel make than the hind wheel in going 52.5^{Km}?

12. How many square decimeters in a rectangular surface 24^{cm} long and 18^{cm} wide?

LI. VOLUME MEASURES

1. A meter cube is 1^{m} long, 1^{m} wide, and 1^{m} high. Then it is ——^{dm} long, ——^{dm} wide, and ——^{dm} high. Hence a cubic meter contains —— cubic decimeters.

2. In linear measures the scale is 10; in surface measures the scale is 100; in volume measures the scale is 1000.

1000 cubic millimeters $^{(cu\ mm)}$ = 1 cubic centimeter $^{(cu\ cm)}$.

1000 cubic centimeters = 1 cubic decimeter $^{(cu\ dm)}$.

1000 cubic decimeters = 1 *cubic meter* $^{(cu\ m)}$.

Copy and fill the blanks:

3. $5^{\text{cu m}} = \_\_\_\_\_^{\text{cu dm}} = \_\_\_\_\_^{\text{cu cm}} = \_\_\_\_\_^{\text{cu mm}}$.

4. $36.4^{\text{cu m}} = \_\_\_\_\_^{\text{cu dm}} = \_\_\_\_\_^{\text{cu cm}} = \_\_\_\_\_^{\text{cu mm}}$.

5. $74{,}632^{\text{cu mm}} = \_\_\_\_\_^{\text{cu cm}} = \_\_\_\_\_^{\text{cu dm}} = \_\_\_\_\_^{\text{cu m}}$.

6. $526.4^{\text{cu mm}} = \_\_\_\_\_^{\text{cu cm}} = \_\_\_\_\_^{\text{cu dm}} = \_\_\_\_\_^{\text{cu m}}$.

In measuring wood the cubic meter is called a *stere* (st), one tenth of a stere is called a *decistere* (dst), and 10 steres are called a *dekastere* (Dst).

7. Find the cost of 17 steres 4 decisteres of wood at $1.75 a stere.

8. If 5^{st} 8^{dst} of wood cost $12.47, what will 17^{st} 4^{dst} cost?

9. How many cubic meters in a rectangular solid 40^{m} long, 8^{dm} wide, and 2^{cm} thick?

LII. CAPACITY MEASURES

1. A vessel that will hold a volume equal to a cubic decimeter has the capacity of one liter.

10 milliliters (ml) = 1 centiliter (cl).
10 centiliters = 1 deciliter (dl).
10 deciliters = 1 *liter* (l).
10 liters = 1 dekaliter (Dl).
10 dekaliters = 1 hektoliter (Hl).
10 hektoliters = 1 kiloliter (Kl).

Copy and fill the blanks:

2. $4^{l} = \_\_\_\_\_^{dl} = \_\_\_\_\_^{cl} = \_\_\_\_\_^{ml}$.

3. $2^{Hl} = \_\_\_\_\_^{l} = \_\_\_\_\_^{ml}$.

4. $826{,}495^{ml} = \_\_\_\_\_^{l} = \_\_\_\_\_^{Hl}$.

5. $7.3^{Kl} = \_\_\_\_\_^{Hl} = \_\_\_\_\_^{l} = \_\_\_\_\_^{dl} = \_\_\_\_\_^{cl} = \_\_\_\_\_^{ml}$.

6. $5.46327^{Kl} = \_\_\_\_\_^{Hl} = \_\_\_\_\_^{Dl} = \_\_\_\_\_^{l} = \_\_\_\_\_^{dl} = \_\_\_\_\_^{cl} = \_\_\_\_\_^{ml}$.

7. $74693.5^{ml} = \_\_\_\_\_^{cl} = \_\_\_\_\_^{dl} = \_\_\_\_\_^{l} = \_\_\_\_\_^{Dl} = \_\_\_\_\_^{Hl} = \_\_\_\_\_^{Kl}$.

8. $8469.5^{ml} = \_\_\_\_\_^{l} = \_\_\_\_\_^{Hl}$.

9. Remember that a liter equals a cubic decimeter. How many liters in a cubic meter?

10. A bin is 3^{m} long, 1.5^{m} wide, and 1.4^{m} high. How many liters will it hold?

11. A bin is 4^{m} long, 2.3^{m} wide, and 3^{m} high. How many hektoliters will it hold?

12. \$12.50 were paid for some peanuts at 5¢ a liter. How many hektoliters were there?

13. Find the cost of a kiloliter of peas at 5¢ a liter.

14. Change to liters and add: 5.2^{Hl}, 4.37^{Dl}, 856^{cl}, 96.7^{ml}, and 84.6^{dl}.

15. From 8.279^{Hl} of corn, 8.64^{Dl} were sold. How many hektoliters were left?

16. From 84.5^{Hl} of wheat, 84.5^{Dl} were sold. What is the value of the rest at \$1.80 per Hl?

LIII. WEIGHT

1. The unit of weight is the *gram*. It is the weight of a cubic centimeter of distilled water.

10 milligrams $^{(mg)}$ = 1 centigram $^{(cg)}$.
10 centigrams = 1 decigram $^{(dg)}$.
10 decigrams = 1 *gram* $^{(g)}$.
10 grams = 1 dekagram $^{(Dm)}$.
10 dekagrams = 1 hektogram $^{(Hg)}$.
10 hektograms = 1 kilogram $^{(Kg)}$.
10 kilograms = 1 myriagram $^{(mg)}$.
10 myriagrams = 1 quintal $^{(Q)}$.
10 quintals = 1 tonneau $^{(T)}$.

The kilogram, about $2\frac{1}{5}$ lb., is more frequently used in ordinary business than any of the other denominations.

Copy and fill the blanks:

2. $3^{T} = \_\_\_\_^{Q} = \_\_\_\_^{Mg} = \_\_\_\_^{Kg} = \_\_\_\_^{Hg} = \_\_\_\_^{Dg} = \_\_\_\_^{g} = \_\_\_\_^{dg} = \_\_\_\_^{cg} = \_\_\_\_^{mg}$.

3. $6^{T} = \_\_\_\_^{Kg} = \_\_\_\_^{g} = \_\_\_\_^{mg}$.

4. $84{,}639{,}457^{mg} = \_\_\_\_^{cg} = \_\_\_\_^{dg} = \_\_\_\_^{g} = \_\_\_\_^{Dg} = \_\_\_\_^{Hg} = \_\_\_\_^{Kg} = \_\_\_\_^{Mg} = \_\_\_\_^{Q} = \_\_\_\_^{T}$.

5. $84{,}639{,}457^{mg} = \_\_\_\_^{g} = \_\_\_\_^{Kg} = \_\_\_\_^{T}$.

6. Since a cubic centimeter of water weighs 1 gram, what is the weight of a cubic decimeter of water?

7. What is the weight of a cubic meter of water in grams? in Kg? in T?

8. Find the cost of a kilogram of quinine at 10¢ a gram.

LIV. PROBLEMS

Equivalents:

1 meter	= 39.37 + in.	1 stere	= 1.308 + cu. yd.
1 sq. meter	= 1.196 + sq. yd.	1 liter	= 1.056 + qt.
1 hektar	= 2.47 + A.	1 gram	= 15.432 + grains.
1 cu. meter	= 1.308 + cu. yd.	1 tonneau	= 2204.6 + lb.

Use the above equivalents in the following problems:

1. How many feet in 9^{m}?

2. Find the cost of 10^{Ha} of land at \$200 an acre.

3. Find the cost of plastering 1000^{qm} at 20¢ a square yard.

4. What will it cost to remove 1000^{m} of earth at 50¢ per cubic yard?

5. Find the value of 2000^{l} of milk at \$.06 per quart.

6. Find the value of 3000^{g} of quinine at $\frac{1}{4}$¢ per grain.

7. How many tons are equal to 4000 tonneaux, or metric tons?

8. How many meters in 1929.13 inches?

9. Find the value of 2148.9 A. at \$320 per hektar.

10. What will it cost to remove 94.176 cu. yd. of earth at \$.75 per cubic meter?

11. Find the cost of 193.208 quarts at 6¢ per liter.

12. What is the value of 77,161 lb. of hay at \$23 per metric ton? (Tonneau.)

13. Find the cost of 4938.24 grains at 5¢ per gram.

LV. SECOND POWER AND SQUARE ROOT

ORAL

1. A 1-foot square contains —— sq. ft.
2. A 2-foot square contains —— sq. ft.
3. A 3-foot square contains —— sq. ft.
4. A 4-foot square contains —— sq. ft.
5. The square of 1 is 1. $1^2 = 1$. $1 \times 1 = 1$.
6. The square of 2 is 4. $2^2 = 4$. $2 \times 2 = 4$.
7. The square of 3 is ——. $3^2 =$ ——. $3 \times 3 =$ ——.
8. The square of 4 is ——. $4^2 =$ ——. $4 \times 4 =$ ——.
9. $5^2 =$ ——. (The square of 5 = 25. $5 \times 5 = 25$.)
10. $6^2 =$ ——.
11. $7^2 =$ ——.
12. $8^2 =$ ——.
13. $9^2 =$ ——.
14. $10^2 =$ ——.
15. $11^2 =$ ——.
16. $12^2 =$ ——.
17. $13^2 =$ ——.
18. $14^2 =$ ——.
19. The square root of 16 is 4. $\sqrt{16} = 4$.
20. The square root of 9 is 3. $\sqrt{9} = 3$.
21. The square root of 4 is 2. $\sqrt{4} = 2$.
22. The square root of 1 is 1. $\sqrt{1} = 1$.
23. $\sqrt{25} =$ ——.
24. $\sqrt{36} =$ ——.
25. $\sqrt{49} =$ ——.
26. $\sqrt{64} =$ ——.
27. $\sqrt{81} =$ ——.
28. $\sqrt{100} =$ ——.
29. $\sqrt{121} =$ ——.
30. $\sqrt{144} =$ ——.
31. $\sqrt{169} =$ ——.
32. $\sqrt{196} =$ ——.
33. $\sqrt{225} =$ ——.
34. $\sqrt{256} =$ ——.

WRITTEN

35. $14^2 =$ ——. 39. $18^2 =$ ——. 43. $22^2 =$ ——.
36. $15^2 =$ ——. 40. $19^2 =$ ——. 44. $23^2 =$ ——.
37. $16^2 =$ ——. 41. $20^2 =$ ——. 45. $24^2 =$ ——.
38. $17^2 =$ ——. 42. $21^2 =$ ——. 46. $25^2 =$ ——.

LVI. SQUARE ROOT (Continued)

Copy and fill the blanks:

1. $86^2 =$ ——. 4. $125^2 =$ ——. 7. $235^2 =$ ——.
2. $845^2 =$ ——. 5. $247^2 =$ ——. 8. $347^2 =$ ——.
3. $748^2 =$ ——. 6. $230^2 =$ ——. 9. $965^2 =$ ——.

Observe that the squares of the above numbers are expressed by twice as many figures as the numbers, or by one figure less than twice as many.

10. $86 = 80 + 6$. The square of $86 = (80 + 6)^2$.

$$\begin{array}{rl} 80+6 & \\ 80+6 & \\ \hline 6400+80\times 6 & =(80+6)\times 80 \\ \quad 80\times 6+36 & =(80+6)\times 6 \\ \hline 6400+2(80\times 6)+36 & =7396 \end{array}$$

11. $845 = 840 + 5$. The square of $845 = (840 + 5)^2$.

$$\begin{array}{rl} 840+5 & \\ 840+5 & \\ \hline 705600+840\times 5 & =(840+5)\times 840 \\ \quad 840\times 5+25 & =(840+5)\times 5 \\ \hline 705600+2(840\times 5)+25 & =714025 \end{array}$$

Express in units of the 1st and 2d orders and solve as the 10th and 11th are solved:

12. 125^2.	**15.** 235^2.	**18.** 845^2.
13. 247^2.	**16.** 347^2.	**19.** 673^2.
14. 230^2.	**17.** 965^2.	**20.** 804^2.

It is evident from Exs. 10–20 that the square of a number expressed by more than one figure is equal to the square of the units of the 2d order (taking the 10th, the square of 80) + twice the product of the units of the 1st and 2d orders (2 times [80 × 6]) + the square of the units of the 1st order (36).

LVII. SQUARE ROOT. METHODS

1. Find (or extract) the square root of 625.

$$\begin{array}{r} 6'25)\underline{25} \\ \underline{4} \\ 45)\underline{225} \\ \underline{225} \end{array}$$

Beginning at the units of the first order, make as many periods of two figures each as possible. The largest perfect square less than 6 is 4. Write the 4 under 6 and its square root, 2, as the first figure of the square root sought. Subtract 4 from 6 and bring down the next period, 25. We have now subtracted 400 (the square of 20) from 625. Take twice 2 tens for a trial divisor. We omit the cipher, and divide 225, *exclusive* of the 5 by 4. We obtain 5 for the second figure of the root. Annex it also to the trial divisor, 4 (add it to 4 tens). The 45 is twice the tens plus the units of the square root. Multiply 45 by 5 and write the result under 225.

Second Method

$$\sqrt{625} = \sqrt{5 \times 5 \times 5 \times 5} = 5 \times 5 = 25.$$

The second method, by factoring, is possible when the number is a perfect square.

Extract the square root of:

2. 676.
3. 729.
4. 784.
5. 841.
6. 696.96.
7. 772.84.
8. 8.0089.
9. 84100.

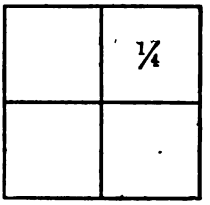
$$\begin{array}{r} 6'96.'96)\underline{26.4} \\ \underline{4} \\ 46)296 \\ \underline{276} \\ 524)2096 \\ \underline{2096} \end{array}$$

LVIII. SQUARE ROOT. FRACTIONS

1. Extract the square root of $\frac{1}{4}$.

This means, what part of *one side* of a square is the *side* of a square $\frac{1}{4}$ as large?

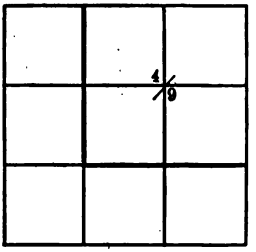
We see by the illustration that it is $\frac{1}{2}$ as large; hence the square root of $\frac{1}{4}$ is $\frac{1}{2}$.



2. Extract the square root of $\frac{4}{9}$.

This means, what part of one side of a square is the side of a square $\frac{4}{9}$ as large?

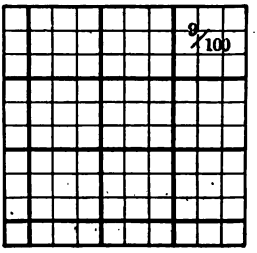
We see by the illustration that it is $\frac{2}{3}$ as large; hence the square root of $\frac{4}{9}$ is $\frac{2}{3}$.



3. Extract the square root of .09.

This means, what part of one side of a square is the side of a square .09 as large?

We see by the illustration that it is .3 as large; hence the square root of .09 is .3.



Note that the large square is divided into 100 equal squares. 9 of these small squares make a square which is .09 of the large square, and one side of this square is 3 units of the 10 units composing a side of the large square; hence .3 as large.

Extract the square root of the following by making diagrams as shown above, after writing what each example means:

4. $\frac{1}{9}$. 7. $\frac{4}{25}$. 10. .01. 13. .36.

5. $\frac{1}{16}$. 8. $\frac{1}{36}$. 11. .25. 14. .49.

6. $\frac{1}{25}$. 9. $\frac{4}{49}$. 12. .16. 15. .64.

LIX. SQUARE ROOT. PROBLEMS

1. $.5^2 =$ ——. $.9^2 =$ ——. $.1^2 =$ ——. $.2^2 =$ ——.

When we square tenths, we obtain ——ths; therefore, to extract the square root of tenths, first change the tenths to hundredths.

2. Extract the square root of .1.

$$\begin{array}{r} .10'00'00)\underline{.316+} \\ 9 \\ \hline 61)100 \\ 61 \\ \hline 626)3900 \\ 3756 \\ \hline 144 \end{array}$$

In both whole numbers and decimals we begin at the decimal point to point off into periods.

$$\begin{array}{r} .20'00'00)\underline{.447+} \\ 16 \\ \hline 84)400 \\ 336 \\ \hline 887)6400 \\ 6209 \\ \hline 191 \end{array}$$

Find the square root to 3 decimal places:

3. $\sqrt{.4}$.	6. $\sqrt{.5}$.	9. $\sqrt{7.3}$.	12. $\sqrt{\frac{7}{8}}$.
4. $\sqrt{.2}$.	7. $\sqrt{1.3}$.	10. $\sqrt{4.5}$.	13. $\sqrt{\frac{3}{4}}$.
5. $\sqrt{.3}$.	8. $\sqrt{2.2}$.	11. $\sqrt{\frac{2}{3}}$.	14. $\sqrt{\frac{5}{6}}$.

(Change first to a decimal of 6 places.)

15. $\sqrt{\frac{4}{9}} =$ ——. When both terms of a fraction are perfect squares, find the square roots of both. In other cases (see 11–14), first change to decimals.

16. A square field contained 60.84 sq. rd. Find the length of one side.

17. A certain square has an area of $\frac{16}{25}$ sq. ft. How long is one of its sides?

18. A certain square field contained 10 A. Find the length of one side in rods.

19. A rectangular field containing 40 A. is 40 rd. wide. How many rods in the perimeter?

20. How many rods in the perimeter of a square field of 40 A.?

LX. POWERS. CUBE ROOT (*a*)

1. $2 \times 2 \times 2 = 8$. This may be written, $2^3 = 8$.
   2^3 is read "the cube of 2."
   $3 \times 3 \times 3 = 27$. This may be written $3^3 = 27$.
   8 is the cube of 2. 27 is the cube of 3.
   2 is the cube root of 8. 3 is the cube root of 27.
   2^3 may also be read "the third power of 2."
   3^3 may also be read "the third power of 3."

2. The cubes of

1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.

3. The cube roots of

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000,
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

4. Learn Exs. 2 and 3, and write them from memory.

5. The cube root of 27 is written $\sqrt[3]{27}$.
The cube root of 64 is written $\sqrt[3]{64}$.

Find the values of the following expressions:

6. 7^2.	14. $\sqrt[3]{216}$.	22. $.2^3$.	29. $(\frac{4}{5})^3$.
7. 7^3.	15. $\sqrt[3]{729}$.	23. $.6^3$.	30. $(\frac{5}{6})^3$.
8. 7^4.	16. 1.6^3.	24. 1.3^3.	31. $.01^3$.
9. 7^5.	17. 2.4^3.	25. 7.5^3.	32. $.02^3$.
10. $\sqrt[3]{8}$.	18. 3.2^3.	26. $(\frac{2}{3})^3$.	33. $.03^3$.
11. $\sqrt[3]{1000}$.	19. $.8^3$.	27. $(\frac{3}{4})^3$.	34. $.04^3$.
12. $\sqrt[3]{125}$.	20. $.5^3$.	28. $(\frac{7}{8})^3$.	35. 1.03^3.
13. $\sqrt[3]{343}$.	21. $.1^3$.		

LXI. CUBE ROOT (*b*)

Show that the following statements are true:

1. $\sqrt[3]{125} = 5$.
2. $\sqrt[3]{729} = 9$.
3. $\sqrt[3]{2179} = 13$.
4. $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$.
5. $\sqrt[3]{.008} = .2$.
6. $\sqrt[3]{.027} = .3$.

7. $\sqrt[3]{\frac{125}{216}} = \frac{5}{6}$.

8. $\sqrt[3]{\frac{343}{512}} = \frac{7}{8}$.

9. $\sqrt[3]{\frac{27}{64}} = \frac{3}{4}$.

10. $\sqrt[3]{\frac{512}{1728}} = \frac{2}{3}$.

11. $(\frac{7}{11})^3 = \frac{343}{1331}$.

12. $(\frac{5}{12})^3 = \frac{125}{1728}$.

13. $(\frac{9}{10})^3 = \frac{729}{1000}$.

14. $(\frac{11}{12})^3 = \frac{1331}{1728}$.

15. $(\frac{5}{13})^3 = \frac{125}{2197}$.

16. What is the length of one edge of a cube containing 125 cu. in.? What is the area of one of its faces?

17. Find the area of one face of a cube containing 64 cu. in.

18. Find the area of one face of a cube containing 343 cu. in.

19. The edge of a cube is 3 in. What is the area of all its faces? What is the volume of the cube?

20. One face of a cube contains 81 sq. in. How long is one of its edges? What is its volume?

21. If the six faces of a cube contain 54 sq. in., what is the volume?

22. The ratio of the volumes of two cubes is 8 : 27. What is the ratio of their edges?

23. The ratio of the edges of two cubes is 3 : 4. What is the ratio of their volumes?

24. The volumes of two cubes are as 64 : 125. What is the ratio of their edges?

LXII. CUBE ROOT (*c*)

1. Write the cubes of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

2. From your answers to Ex. 1 state how many figures in the cubes of numbers of 1 figure.

3. Write the cubes of 13, 16, 56, 85, 98.

4. From your answers to Ex. 3 state how many figures in the cubes of numbers of 2 figures.

5. In square root, how many figures form a period?

6. How many figures are there in the cube root of 729? 27? 8?

7. Judging from your answers in Ex. 1, how many figures in the cube root of a number of 1, 2, or 3 figures?

8. Write the cubes of 10, 20, 30, 40, 50, 60, 70, 80, 90.

9. Judging from your answers in Ex. 8, how many figures are there in the cube root of a number of 4, 5, or 6 figures?

10. What is the first figure in the cube root of 1331? 2197? 9261? 592,704? (See Ex. 8.)

11. Having determined the first figure of the cube root of each of these numbers, 1331, 2197, 9261, and 592,704 in Ex. 10, find the second figure by seeing which of the first nine natural numbers cubed will give the right-hand digit.

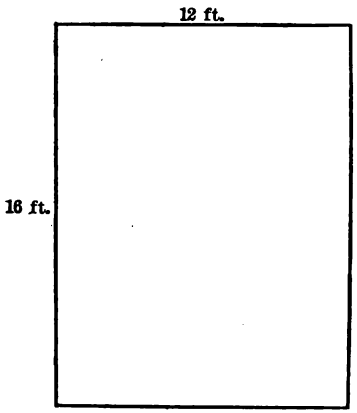
SUGGESTION. — Taking 9261: $10^3 = 1000$, $20^3 = 8000$, while $30^3 = 27{,}000$; the first figure of the cube root is evidently 2. Then $1^3 = 1$, $2^3 = 8$, etc. We see that the only number of one figure which gives 1 when cubed is 1. Therefore the cube root of 9261 is 21. Verify this.

12. In like manner find the cube root of the following perfect cubes and verify: 1728, 2744, 4913, 3375, 6859, 4096, 5832, 421,875, 614,125, 941,192, 970,299, 15,625, 17,576.

NOTE. — For further treatment, see Appendix.

LXIII. MENSURATION. CARPETING (*a*)

(Use 1-foot rules.)



1. Draw a diagram of a room 12 feet wide and 16 feet long to the scale of $\frac{1}{8}$ inch to the foot. Your drawing is —— inches wide and —— inches long.

2. In carpeting a floor the carpet is unrolled and cut into lengths long enough to reach across the room or lengthwise of the room. These are sewed together. How many feet in 1 strip across the above room?

3. How many strips will be needed if the carpet is 1 yard wide? (There being no waste in cutting.)

4. How many feet in width of the last strip must be cut off or turned under?

5. How many feet in length of the roll must be purchased to carpet the room?

6. Find the cost of the carpet at $1.50 a yard.

7. If there is a waste of 6 inches on each strip after the first one in matching the pattern, how many feet of carpet must be bought?

8. Find the cost of the carpet at $1.25 a yard.

9. If the strips are laid lengthwise of the room, how many strips will be required?

10. If there is no waste in matching the pattern, how many feet in length of carpet must be bought?

11. Find the cost at $1.75 a yard.

LXIV. MENSURATION. CARPETING (*b*)

1. Draw a diagram of a room 15 ft. wide and 20 ft. long to the scale of $\frac{1}{8}$ of an inch to the foot.

2. Your drawing is —— inches wide and —— inches long.

3. How many strips of carpet will be needed for this room, the strips to run lengthwise of the room and the carpet being $\frac{3}{4}$ yd. wide?

4. How many feet in width of the last strip must be cut off or turned under?

5. If there is no waste in matching the pattern, how many feet in length of carpet must be purchased?

6. What is the value of the carpet laid, the cost price being $1.25 per yard, and a charge of 10¢ a yard being made for laying?

7. If there is a waste of 6 in. for matching on each strip after the first one, what will the carpet cost? (Not laid.)

8. Make a diagram of a room 18 ft. by 22 ft., scale $\frac{1}{8}$ in. to the foot, and find the cost of carpeting this room with carpet 1 yd. wide, the carpet to run lengthwise, the cost price being 75¢ a yard. (No loss in matching.)

9. How much more would it have cost if the strips had run crosswise? Note the waste due to turning under part of the last strip.

LXV. BOARD MEASURE

1. A board, not more than one inch thick, containing on one side an area of 144 sq. in. contains one board foot.

2. Lumber is bought and sold by the board foot.

3. A piece of lumber 1 ft. long, 1 ft. wide, and 1 in. thick contains 1 board foot. A piece 12 ft. long, 1 in. wide, and 1 in. thick contains 1 board foot. A piece 12 ft. long, 2 in. wide, and 1 in. thick contains 2 board feet.

4. A piece of lumber 12 ft. long, 3 in. wide, and 1 in. thick contains —— board feet.

5. A piece of lumber 12 ft. long, 4 in. wide, and 1 in. thick contains —— board feet.

6. It is evident that lumber 12 ft. long and 1 in. thick contains as many board feet as it is inches wide.

7. How many board feet in a board 12 ft. long, 6 in. wide, and 1 in. thick?

8. How many board feet in 25 boards, each 12 ft. long, 10 in. wide, and 1 in. thick?

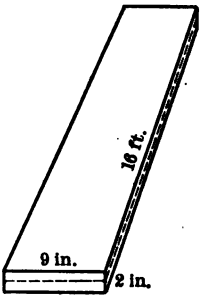
9. How many board feet in 80 boards, each 15 ft. long, 8 in. wide, and 1 in. thick?

NOTE. — First find the result as if they were 12 ft. long. Since 15 equals 12 plus $\frac{1}{4}$ of 12, 15-foot boards will contain $\frac{1}{4}$ as many board feet more than 12-foot boards.

LXVI. BOARD MEASURE (Continued)

1. A piece of lumber 12 feet long, 9 in. wide, and 1 in. thick contains —— bd. ft. (board feet = bd. ft.). 16 ft

being 4 ft. more than 12 ft., that is $\frac{4}{12}$ or $\frac{1}{3}$ of 12 ft. more than 12 ft., we increase our answer by $\frac{1}{3}$ of itself and obtain 12 bd. ft. A piece of lumber 16 ft. long, 9 in. wide, and 2 in. thick will contain 2 times 12 bd. ft., or 24 bd. ft.



A TABLE. LUMBER MEASURE

SIZE	12	14	16	18	20	22	24	26
2 x 4	8	9	11	12	13	15	16	17
2 x 6	12	14	16	18	20	22	24	26
2 x 8	16	19	21	24	26	29	32	35
2 x 10	20	23	27	30	33	37	40	43
2 x 12	24	28	32	36	40	44	48	52
3 x 6	18	21	24	27	30	33	36	39
3 x 8	24	28	32	36	40	44	48	52
3 x 10	30	35	40	45	50	55	60	65
3 x 12	36	42	48	54	60	66	72	78

2. Study the first horizontal line. A board 12 ft. long and 4 in. wide contains 4 bd. ft.; the piece of lumber, or joist, contains 2×4 bd. ft., or 8 bd. ft.

3. A board 14 ft. long contains $\frac{1}{6}$ of 8 bd. ft. more lumber than a board 12 ft. long. $\frac{1}{6}$ of 8 is $1\frac{1}{3}$. As $\frac{1}{3}$ is less than $\frac{1}{2}$, it is dropped, and 1 is added to 8, making 9 bd. ft.

4. A board 16 ft. long contains $\frac{1}{3}$ of 8 bd. ft. more lumber than a board 12 ft. long. $\frac{1}{3}$ of 8 is $2\frac{2}{3}$. As $\frac{2}{3}$ is more than $\frac{1}{2}$, it is considered as 1, and $2\frac{2}{3}$ becomes 3. 3 is added to 8, making 11.

In these calculations $\frac{1}{2}$, or more, is regarded as 1 whole and less than $\frac{1}{2}$ is dropped.

5. Find the cost of 300 joists, each 16 ft. long, 2×6 (read 2 by 6), at \$18 per thousand board feet. (Refer to the table.)

6. What must I pay for 75 planks, each 20 ft. long, 2×10, at \$20 per thousand? (Per thousand bd. ft.)

A bunch of lath 4 ft. long containing 100 pieces covers 45 sq. yd.

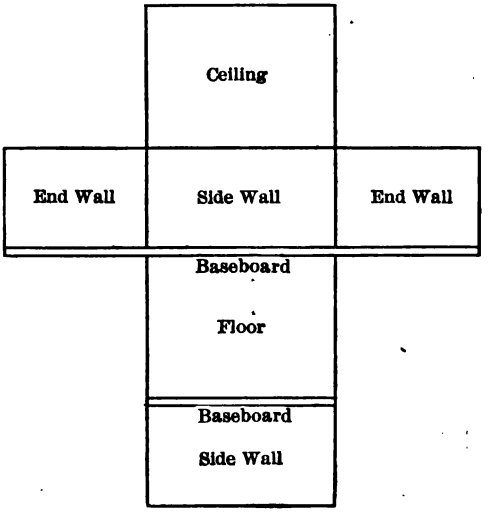
1000 shingles laid 4 in. to the weather will cover 100 sq. ft.

7. How many bunches of lath will be needed to lath the ceiling and walls of a room 12 ft. long, 9 ft. wide, and 8 ft. high?

8. How many shingles will cover a roof 50 ft. long, 15 ft. wide?

LXVII. MENSURATION. ROOMS (*a*)

1. A certain room is 12 ft. wide, 16 ft. long, and 9 ft. high. We have in the diagram a representation of this room. This diagram is called a *development* of the room. The walls, ceiling, and floor are all represented on a flat surface. After careful inspection, draw this *development* of the room on the scale of $\frac{1}{16}$ in. to the foot.



2. Find the cost of plastering the ceiling at \$.35 per square yard.

3. As the room is 12 ft. by 16 ft., the perimeter of the room is how many feet? The 4 walls then are equal to one wall how many feet long? As the walls are 9 ft. high, the area of the walls is how many square feet? Find the cost of plastering the walls at \$.35 per square yard.

4. What is the cost of lathing the walls and ceiling of this room at 4¢ per square yard?

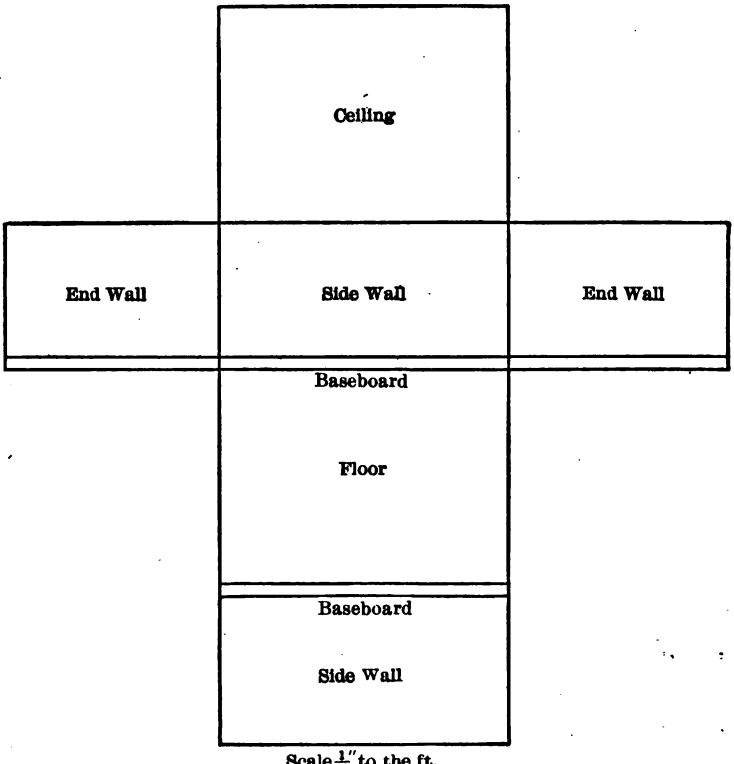
5. How much will the baseboard of this room cost at 4¢ a running foot? (A running foot is a foot in length, regardless of the width.)

6. What would be the cost of this baseboard if there be an allowance for 3 doors, each 2 ft. 6 in. wide?

LXVIII. MENSURATION. ROOMS (*b*)

1. This diagram is the development of a room drawn to the scale of $\frac{1}{16}$ in. to the foot.

How long is the room?



2. How wide is the room?

3. How high?

4. How long is the baseboard?

5. Find the cost of plastering the walls and ceiling at 40¢ per square yard.

6. Find the cost of the baseboard at 5¢ per running foot, no allowance being made for doors.

7. Find the cost of covering the floor with oilcloth at 50¢ per square yard.

8. Find the cost of carpeting the floor with carpet $\frac{3}{4}$ yd. wide, the strips running lengthwise, there being no loss in matching, and the price being 95¢ a yard.

LXIX. MENSURATION. ANGLES. POLYGONS

1. The rectangle, with which we are familiar, has square corners, corners which are called *right angles*. Now we are to consider figures whose corners are not all right angles. The angles with which we have to do are the three which follow.

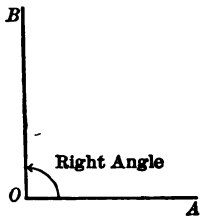


FIG. 1.

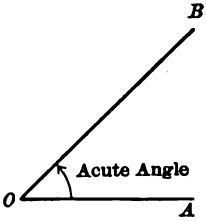


FIG. 2.

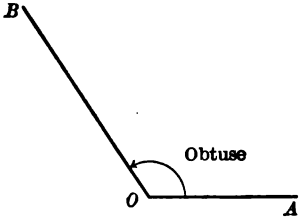


FIG. 3.

2. Suppose BO to revolve around the point O in the direction shown by the arrow. BO would form with the line AO various angles. The greater the divergence, or

difference in direction of the lines, the greater the angle. The size of the angle depends in no wise upon the length of its sides, but entirely upon the amount of difference in direction. Until BO reaches the position with reference to AO shown in Fig. 1, angle AOB is an acute angle. An acute angle, then, is less than a right angle.

When BO revolves beyond the position shown in Fig. 1, it forms with AO an obtuse angle, as shown in Fig. 3. An obtuse angle, then, is greater than a right angle.

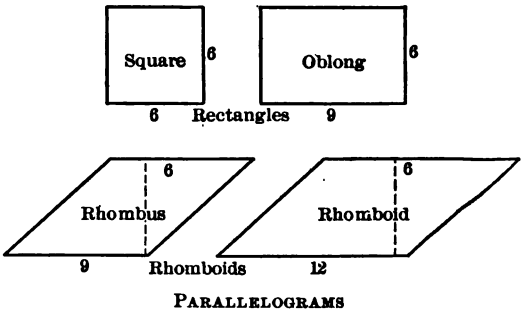
If BO were prolonged through O (Fig. 1), it would form with AO another equal angle below AO.

But prolonging BO through O in Fig. 2 or in Fig. 3 would not form an equal angle.

A right angle can always be determined by prolonging either side through the point of meeting, called the *vertex* of the angle. If the two angles are equal, they are right angles.

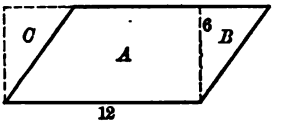
The two sides of a right angle are said to be perpendicular to each other. A line is perpendicular to another line when the two form a right angle.

3. We already know how to find the area of rectangles. In the square above 6 is the base and 6 is the altitude. The area $= 6 \times 6 = 36$.



Let us study the other kind of parallelogram, the rhomboid.

In the rhomboid A we may form the triangle B by drawing the altitude at the right end of the base. Now think of the triangle B as moved to the left until it occupies the position of triangle C. Then we evidently have a *rectangle* of the same base and altitude as the rhomboid, and of the same area. The method of finding the area of a parallelogram is evidently the same as the method of finding the area of a rectangle.



4. This parallelogram represents a plot of ground drawn to the scale of $\frac{1}{8}$ in. to the foot. Find the area of the plot.

5. What is the area of the rhombus given above? The area of the rhomboid?

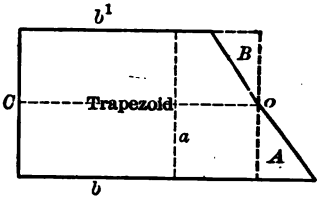
6. A parallelogram has its opposite sides parallel. If its angles are right angles, the figure is a rectangle. If two angles are acute and the other two are obtuse, the figure is a rhomboid.

LXX. MENSURATION. POLYGONS

1. Polygons are figures bounded by straight lines. Polygons of three sides are *triangles*. Polygons of four sides are *quadrilaterals*. Polygons of five sides are *pentagons*. Polygons of six sides are *hexagons*.

2. It will be remembered that parallelograms have been considered. We have two other polygons of four sides yet to consider.

3. How many sides of this polygon are parallel? A quadrilateral having only two sides parallel is a trapezoid. By drawing a dotted line through the mid-point of one non-parallel side parallel to the opposite side, and then producing the upper base, two equal triangles are formed.

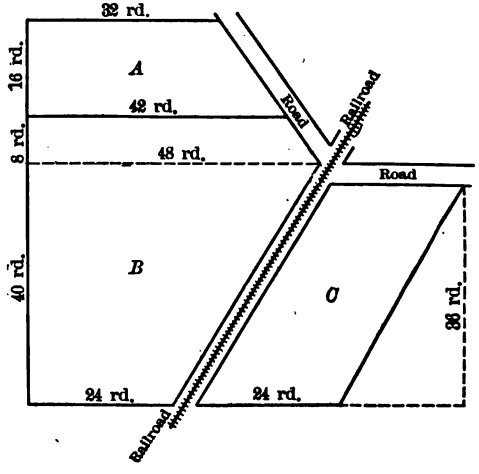


Substituting triangle A in place of triangle B, a parallelogram is formed equal to the original trapezoid.

The upper and lower sides of the parallelogram are evidently equal, and each is equal to CO. But the upper base of the trapezoid was lengthened as much as the lower base was shortened. Hence their sum equals the sum of the two bases of the parallelogram.

Therefore half the sum of the bases of the trapezoid equals CO. Hence the area of the trapezoid equals half the sum of the parallel sides multiplied by the altitude.

4. Fields A, B, and C border on two roads, as shown. The dotted lines indicate the additional measurements desired for solution.

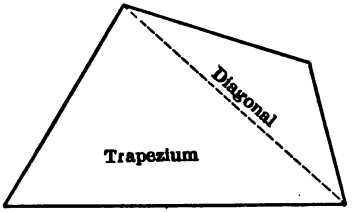


What is the form of field A? Of field

B? How long is the base of field C? How great is the altitude? What is the form of field C?

5. How many acres in A? In B? In C?

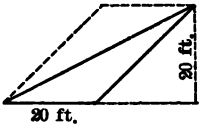
6. How many sides has this figure? No sides are parallel. A quadrilateral with no sides parallel is a trapezium.



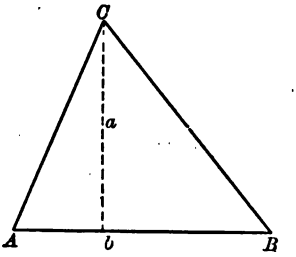
7. Quadrilaterals are, then, divided into three classes, viz.: parallelograms, trapezoids, and trapeziums.

8. By drawing a diagonal we form two triangles. It is evident that the trapezium equals the sum of the two triangles.

9. Compare the area of the triangle with the area of the parallelogram. The area of the parallelogram is ——. The area of the triangle is ——.



10. In triangle ABC the base b is 33 ft. and the altitude a is 27 ft. What is the area?



11. The base of a triangle is 104 yd., and its altitude is 30 yd. Find its area.

LXXI. MENSURATION. PARALLELOGRAMS. TRIANGLES

1. This triangle is called a *right triangle* because it has one right angle. The side c opposite the right angle is called the *hypotenuse.* The sides a and b are called *legs of the right triangle.*

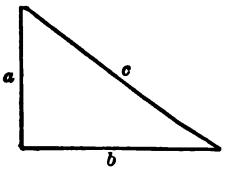


FIG. 1.

2. In Fig. 2, suppose leg $a=3$ in., leg $b=4$ in., and the hypotenuse $c=5$ in. By drawing the squares of a, b, and c as shown, and dividing them into 1-inch squares, we see that $a^2+b^2=c^2$; for, substituting the values of a, b, and c in place of these letters, we have $9+16=25$.

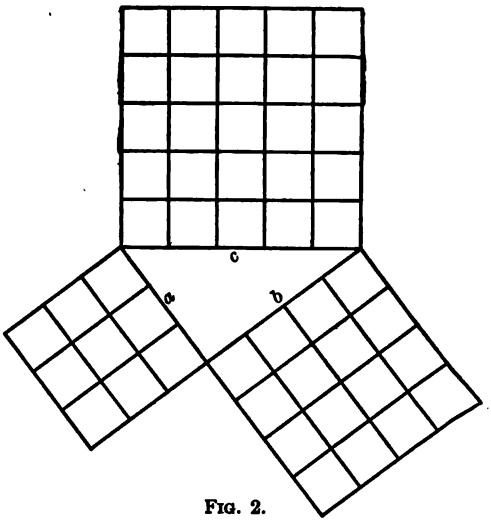


FIG. 2.

3. The square of the hypotenuse equals the sum of the squares of the two legs.

Therefore, to find the hypotenuse, extract the square root of the sum of the squares of the legs.

4. How long a ladder will reach a window 12 ft. from the ground, if the foot of the ladder is 9 ft. from the building?

5. If the hypotenuse of a right triangle is 25 ft. and one of the legs 20 ft., how long is the other leg?

6. How long is the diagonal (the distance from one corner to the opposite corner) of a 10-ft. square? (Correct to 2 decimal places.)

7. Find the area of a parallelogram whose base is 18 ft. and whose altitude is 14 ft.

LXXII. MENSURATION. TRIANGLES

1. When the three sides of a triangle are given, to find the area, subtract each side separately from half the sum of the sides. Find the product of the three remainders and the half sum. Extract the square root of the product. This method is found in geometry.

2. Find the area of a triangle whose sides are 9 ft., 12 ft., and 15 ft.

$\frac{9+12+15}{2} = 18$, half of the sum of the sides.

$$\left.\begin{aligned} 18-\ 9&=9\\ 18-12&=6\\ 18-15&=3 \end{aligned}\right\}$$ the three remainders.

$\sqrt{9\times6\times3\times18} = 54$. $\therefore$ area $= 54$ sq. ft.

3. Find the area of a triangle whose sides are 6 ft., 8 ft., and 10 ft.

4. Find the area of a triangle whose sides are 4 ft., 7 ft., and 9 ft. (To 2 decimal places.)

5. Find the area of a parallelogram whose base is 56 rd. and whose altitude is 45 rd.

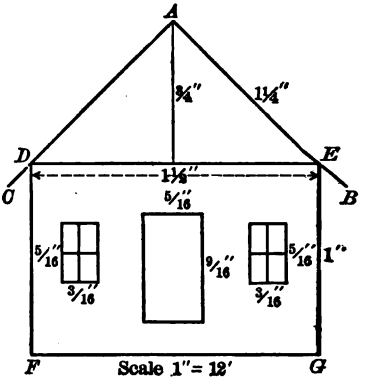
6. Find the area of a triangle whose base is 64 rd. and whose altitude is 28 rd.

7. The base of a triangle is 40 ft. and the area is 320 sq. ft. What is the altitude?

LXXIII. SCALE COMPUTATION

1. Note the scale, 1 in. to 12 ft., and find the lengths of the dimensions:

(1) Width FG; (2) the height to the eaves FD; (3) the rise AO; (4) the length of the rafters AB; (5) the width of the door; (6) the height of the door; (7) the width of each window; (8) the height of each window.



2. How many square feet in the end of this building below the gable, *i.e.* below the line DE, including the door and windows?

3. What is the area total of the door and windows?

4. What is the area of the gable?

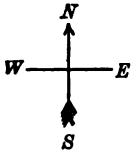
5. What is the total area of the end of this building *exclusive* of the windows and door?

6. How much will the boards for siding the end of this building cost at $35 per M, allowing for the windows and

15 ft., board measure, for waste in fitting? ($35 per M means $35 per thousand feet, board measure.) The boards are 1 in. thick.

7. Make an enlarged drawing of the end of this house to a scale 4 times the scale used here.

LXXIV. TOWNSHIP



6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

A Township. — 36 sections.

1. In many states land is divided into townships, each 6 mi. square. Each township is divided into 36 sq. mi., and each square mile is called a section. The 36 sections of a township are represented in the drawing above. All the straight lines represent the public roads or highways running between the sections.

2. How many miles from the N.W. (northwest) corner of section 6 to the S.E. (southeast) corner of section 31, taking the shortest course by the highway?

3. How many miles from the N. E. (northeast) corner of section 6 to the S. W. (southwest) corner of section 22?

4. How many miles from the N. W. corner of section 6 to the S. E. corner of section 36?

5. How many miles from the S. E. corner of section 31 to the N. W. corner of section 13?

6. How many miles of road are represented in the drawing?

LXXV. SECTION

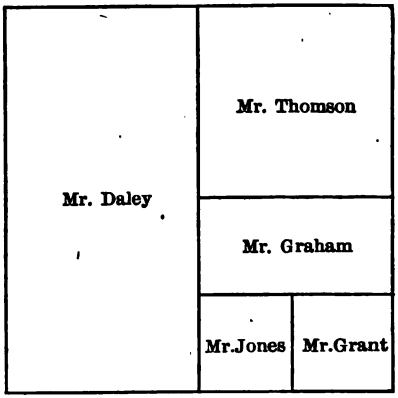
1. Each section of a township is 1 mi. square and contains 640 A. How many acres in Mr. Daley's farm?

2. Mr. Thomson owns the N. E. $\frac{1}{4}$ section. How many acres?

3. Mr. Graham owns the N. $\frac{1}{2}$ of the S. E. $\frac{1}{4}$ section. How many acres?

4. Mr. Jones owns the S. W. $\frac{1}{4}$ of the S. E. $\frac{1}{4}$ section. How many acres?

5. Mr. Grant owns the S. E. $\frac{1}{4}$ of the S. E. $\frac{1}{4}$ section. How many acres?



A SECTION.—640 A.

6. How many rods from the N. W. corner of Mr. Daley's farm to the S. E. corner of Mr. Graham's farm, measuring by the boundaries?

7. How many rods from the N. E. corner of Mr. Thomson's farm to the N. E. corner of Mr. Jones's farm?

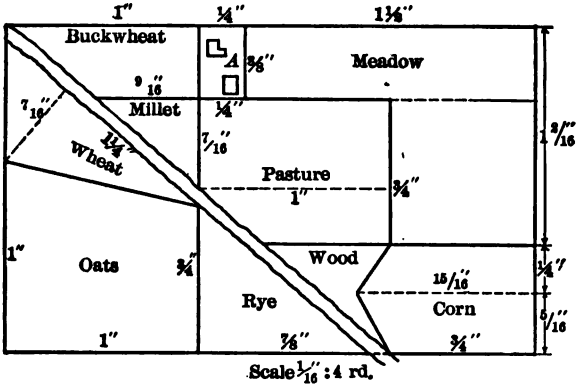
8. What fraction of the section does Mr. Graham own?

9. What fraction of the section does Mr. Grant own?

10. How many rods of fence are necessary to inclose Mr. Daley's farm?

11. How many rods of fence are necessary to inclose Mr. Graham's farm?

LXXVI. ACREAGE



1. *A* is the portion of a farm containing the house and the barn. Note the scale of the drawing. How many rods long is *A*? How many rods wide? How many acres in the lot occupied by the house and barn?

2. The dotted lines are for use in making the computations. How many acres of buckwheat are there?

3. How many acres of millet are there on this farm?

4. How many acres of pasture are there on this farm?

5. How many acres of corn are there on this farm?

6. How many acres of meadow are there on this farm?

7. How many acres of wheat are there on this farm?

8. How many acres of oats are there on this farm?

9. How many acres of rye are there on this farm?

LXXVII. MENSURATION. THE CIRCLE (*a*)

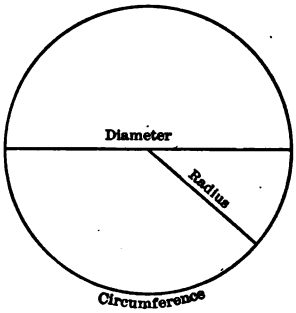


FIG. *a*.

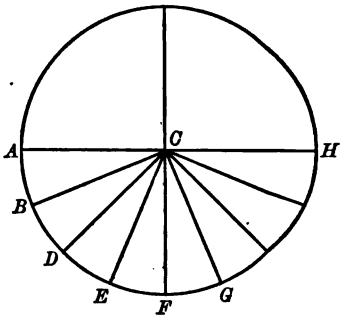


FIG. *b*.

1. The diameter, radius, and circumference of a circle are shown in Fig. *a*. The circumference bounds the circle.

2. The diameter passes through the center and ends in the circumference.

3. The radius measures the distance from the center to the circumference. Compare the radius and the diameter as to length.

4. Cut a circle out of stiff paper or a piece of cardboard. Make it as large as practicable. Divide the circle into halves, and divide each half as the lower shown in Fig. *b* is divided, making arcs *AB*, *BD*, *DE*, *EF*, *FG*, etc., all equal and as small as practicable. Then cut from the center along line *CB*, but not quite through to *B*; like-

wise along lines *CD*, *CE*, *CF*, etc. Then spread the one half open as shown by the darkened portion of Fig. *c*. Cut the other half in a similar manner, and opening it, fit it carefully with the first half. The second half will show as the lighter portion of Fig. *c*. The two portions thus fitted will form nearly a rectangle as long as half the circumference of the circle and as wide as the radius. The smaller the arcs, the more nearly is the figure a rectangle.

5. The area of a circle, then, evidently equals one half of the circumference multiplied by the radius.

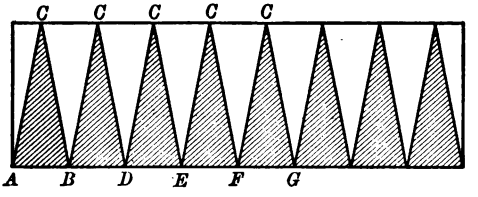


FIG. c.

6. The circumference of a circle is about 3.1416 times the diameter. This is shown in geometry. 3.1416 is about $3\frac{1}{7}$, for .1416 multiplied by 7 equals .9912, which is almost 1. $3\frac{1}{7}$ may be used in the following examples.

7. If the diameter of a circle is 7 inches, its circumference is —— inches.

8. If the circumference of a circle is 66 in., its diameter is —— in.

9. If the circumference of a circle is 88 yd., its diameter is —— yd.

10. If the diameter of a circle is 35 yd., its circumference is —— yd.

11. If the circumference of a wheel is 6 ft., in making one complete turn, it will roll —— ft.

12. A wheel 7 ft. in diameter is —— ft. in circumference. If allowed to roll till it makes 3 revolutions (turns), it will roll —— ft.

13. A wheel 6 ft. in circumference will make —— revolutions in rolling 54 ft.

WRITTEN

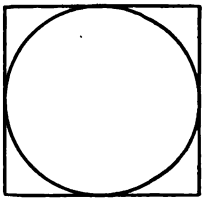
14. Find the circumference of a circle 16 ft. in diameter.

15. Find the circumference of a circle 28 ft. in diameter.

LXXVIII. CIRCLES (*b*)

1. The largest circle which can be cut out of a given square is a little more than $\frac{3}{4}$ as large as the square.

2. The area of a 6-inch square is —— sq. in. The largest circle that can be cut out of a 6-inch square is a little more than $\frac{3}{4}$ of —— sq. in., or —— sq. in.



3. It is to be noted that one side of the square equals the diameter of the circle to be cut from it.

Using $\frac{3}{4}$ as the ratio, find the area of a circle whose diameter is:

4. 4 in. **6.** 6 in. **8.** 12 ft. **10.** 4 yd.

5. 8 in. **7.** 10 in. **9.** 9 ft. **11.** 8 yd.

The exact ratio of a circle to the smallest square from which it can be cut is .7854, correct to within 1 ten-thousandth. Using .7854 as the ratio, solve the following examples.

WRITTEN

12. A circular pond is 80 rods across. How many acres in the area?

13. A man has a circular garden whose diameter is 200 feet. What is the area?

14. How many feet of fence will be required to inclose this garden? (L. LXIV.)

15. A cow is tied to a stake by a chain 40 ft. long. What is the area of the surface over which she can graze?

16. Find the circumference of a circle 63 ft. in diameter.

17. Find the diameter of a circle 110 ft. in circumference.

18. Find the diameter of a circle 77 ft. in circumference.

19. If the radius of a circle is $3\frac{1}{2}$ ft., what is the circumference?

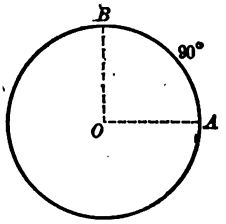
20. If the radius of a circle is $7\frac{1}{2}$ ft., what is the circumference?

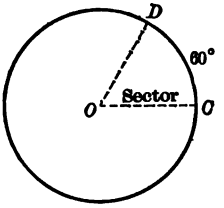
21. If the circumference of a circle is 110 yd., what is the radius?

LXXIX. ARCS OF CIRCLES, ETC.

1. The circumference of every circle is regarded, for convenience, as divided into 360 equal parts called degrees.

2. Any portion of a circumference is called an arc.





3. Since the whole circumference is 360 degrees (360°), the arc AB is an arc of 90°. The angle AOB is an angle of 90°.

4. The arc CD is an arc of 60°. The angle COD is an angle of 60°.

5. The figure bounded by two radii and an arc is called a sector. The sector COD, having an arc of 60°, is $\frac{1}{6}$ of the circle, because a sector has the same ratio to the circle that its arc has to the circumference.

WRITTEN

6. Find the circumference of a circle whose radius is $10\frac{1}{2}$ ft. (L. LXIV.)

7. Find the area of the sector whose arc is 90° in a circle whose diameter is 21 ft.

8. Find the area of the sector whose arc is 60° in a circle whose diameter is 42 ft.

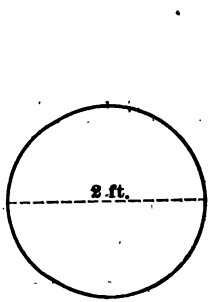
9. Find the circumference of a circle whose diameter is 126 rd.

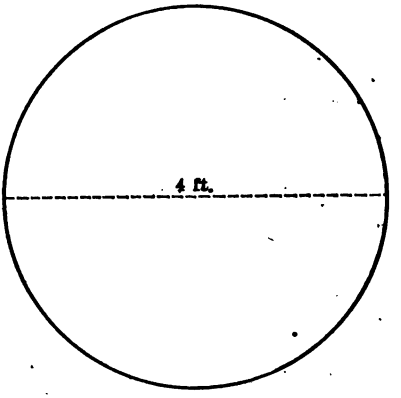
10. Find the diameter of a circle whose circumference is $18\frac{6}{7}$ yards.

11. Find the total area of two circles whose radii are 2 rd. and 3 rd. respectively.

12. Find the radius of a circle whose circumference is 22 yd.

13. Find the area of a circle 2 ft. in diameter.





14. Find the area of a circle 4 ft. in diameter.

15. What is the ratio of the first diameter to the second diameter?

16. What is the ratio of the first area to the second area?

17. Find the area of a circle 6 ft. in diameter.

18. What is the ratio of 2 ft. to 6 ft.?

19. What is the ratio of a circle 2 ft. in diameter to a circle 6 ft. in diameter?

20. The ratio of 2 to 6 is —— ——.

21. The ratio of a circle 2 ft. in diameter to a circle 6 ft. in diameter is —— ——.

22. The ratio of 2 to 8 is —— ——.

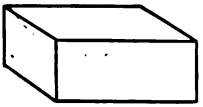
23. The ratio of a circle 2 ft. in diameter to a circle 8 ft. in diameter is —— ——.

24. The ratio of one circle to another equals the square of the ratio of the diameters.

LXXX. SOLIDS

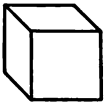
1. A plane, or plane surface, has length and breadth, but no thickness.

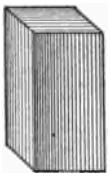
2. A solid, or volume, has the three dimensions of length, breadth, and thickness.



3. A rectangular solid is bounded by 6 rectangles.

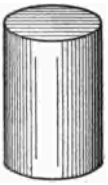
4. A cube is bounded by 6 equal squares.





Quadrangular Prism

Triangular Prism

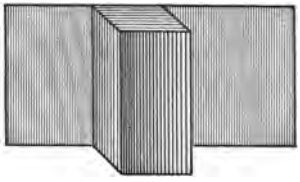


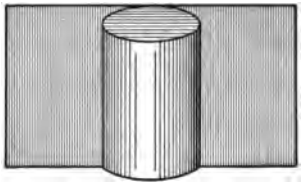
Cylinder

5. A prism has parallelograms for sides, and equal polygons parallel to each other for bases.

6. The bases of a cylinder are equal parallel circles, and the rest of the surface is uniformly curved. The area of the curved surface is called the convex surface.

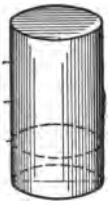
7. The altitude of these figures is the perpendicular distance between their bases.





8. By fitting a piece of paper to the lateral surface of a prism or cylinder, and then unrolling it, it is seen to be a rectangle whose base is equal to the perimeter or circumference of the base, and whose altitude is equal to the altitude of the given figure.

9. It is evident that the volume of a cylinder 4 in. high is 4 times the volume of a cylinder 1 in. high.



10. If the diameter of the base is 3 in., the area of the base is $.7854 \times 9 = 7.0686$ sq. in.

11. The volume of the portion of the cylinder 1 in. high is evidently 7.0686 cu. in. The total volume is 7.0686 cu. in. $\times 4 = 28.2744$ cu. in.

12. Find the convex surface of a triangular prism (a prism whose bases are triangles) whose altitude is 10 ft. and each side of whose base is 4 ft.

13. Find the convex surface of a triangular prism whose altitude is 8 ft. and the sides of whose bases are 3 ft., 4 ft., and 5 ft. respectively.

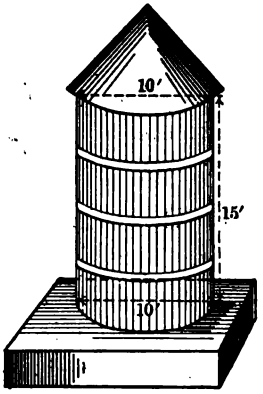
14. Find the *entire* surface of a quadrangular prism whose bases are 3 ft. by 4 ft. and whose altitude is 9 ft.

15. Find the solid contents of a triangular prism, each side of whose base is 6 ft. and whose altitude is 20 ft.

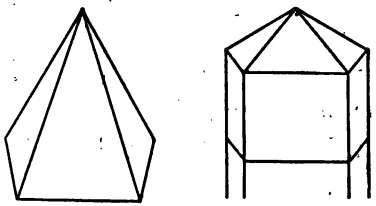
16. How many cubic feet in a cylinder 12 ft. long and 1 ft. in diameter?

17. How many cubic feet must be excavated to form a cylindrical cistern 12 ft. deep and 5 ft. in diameter?

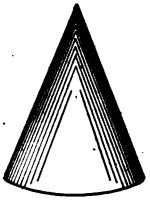
18. How many gallons will a cylindrical tank hold whose diameter is 10 ft. and whose altitude is 15 ft.?



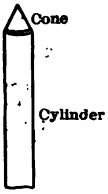
LXXXI. PYRAMIDS AND CONES



Pyramid



Cone



Pencil

1. A pyramid is a solid having a triangle or any other polygon for its base, and triangles ending in a common point for the other faces. The common point is called the vertex.

2. A cone is a solid having a circle for its base, and whose convex surface tapers uniformly to a point. This point is called the vertex.

3. The altitude of a cone or a pyramid is the perpendicular distance from the vertex to the base.

4. If a cone and a cylinder of equal base and altitude be taken, and each be nicely fitted with a sheet of paper wrapped about the curved surface, it can be shown by using sand or other dry substance that the paper cylinder, when removed, will hold three times as much as the paper cone.

5. The slant height of a cone is the distance from the vertex to any point in the circumference of the base.

6. The slant height of a pyramid is the perpendicular distance from the vertex to any side of the base.

7. To find the convex surface of a pyramid or a cone, multiply the perimeter or the circumference of the base by one half of the slant height.

8. To find the solid contents of a pyramid or a cone, multiply the area of the base by one third of the altitude.

9. Find the convex surface of a cone whose slant height is 9 ft. and the diameter of whose base is 5 ft.

$(5^2 \times .7854) \times 9 =$ —— convex surface.

10. Find the convex surface of a pyramid whose base is a 2-ft. square and whose slant height is 5 ft.

$(4 \times 2 \text{ ft.}) \times (\frac{1}{2} \text{ of } 5) =$ —— convex surface.

11. What is the entire surface of a cone whose base is 6 ft. in diameter and whose slant height is 10 ft.?

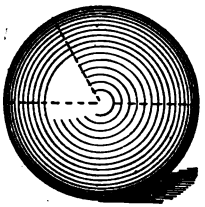
12. What is the entire surface of a pyramid whose base is a 3-ft. square and whose slant height is 12 ft.?

13. Find the solid contents of a cone whose base is 5 ft. in diameter and whose altitude is 6 ft.

14. Find the solid contents of a pyramid whose base is a triangle with sides 21 in., 28 in., and 35 in. long, respectively, and whose altitude is 42 in.

LXXXII. SPHERES

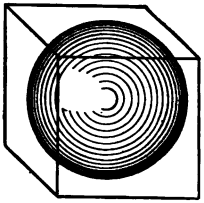
1. A sphere is a solid with a uniformly curved surface, all points of which are equally distant from a point within, called the center.



2. The diameter of a sphere is a straight line which passes through the center and ends in the surface.

3. To find the surface of a sphere, multiply the square of the diameter by 3.1416.

4. A sphere is a little more than one half of the smallest cube from which it can be cut.



5. A sphere is exactly .5238 of the smallest cube from which it can be cut, correct to within one ten-thousandth.

Find the volume (solid contents) of a sphere 5 inches in diameter. The volume of the cube from which it could be cut is $5 \times 5 \times 5 = 125$ cu. in. 125 cu. in. $\times$ .5236 = the volume of the sphere.

6. What is the surface of a sphere whose diameter is 6 inches?

7. Find the volume of a sphere whose diameter is 6 inches.

8. A sphere made of putty is 6 inches in diameter. How many spheres, each with a diameter of 3 inches, could be made from the same putty?

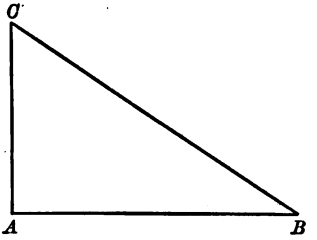
9. What is the ratio of the 6-inch sphere to each 3-inch sphere?

10. What power of the ratio of 6 to 3 equals the ratio of a 6-inch sphere to a 3-inch sphere?

LXXXIII. REVIEW

1. $AC = 75$ yd. $AB = 100$ yd. How long is CB, the hypotenuse?

2. Suppose $AB = 80$ ft. and $CB = 100$ ft. Find the length of AC. (L. LXXI.)

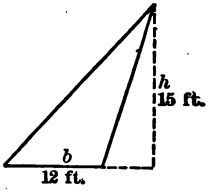


3. Find the diagonal of a 3 ft. square correct to two decimal places.

4. Find the area of a triangle whose sides are 2 yd., 3 yd., and 4 yd., respectively, correct to two decimal places. (L. LXXII.)

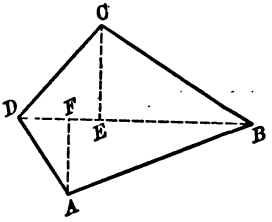
5. Find the area of a parallelogram whose base is 28 rd. and whose altitude is 24 rd. (L. LXIX.)

6. Find the area of this triangle: base $b = 12$ ft., altitude $h = 15$ ft. (L. LXX.)



7. Find the area of a trapezoid whose parallel sides are 80 ft. and 60 ft. respectively, and the perpendicular distance between them is 46 ft. (L. LXX.)

8. Find the area of this trapezium: The diagonal $DB = 200$ ft., the perpendicular $CE = 80$ ft., and the perpendicular $AF = 75$ ft. (L. LXX.)



9. What is the circumference of a circular room which is 20 ft. across? (L. LXXVII.)

10. What is the area of the floor of this circular room? (L. LXXVIII.)

11. The circumference of a wheel is $12\frac{4}{7}$ ft. About how long is one of the spokes?

12. The diameter of a wheel is $3\frac{1}{2}$ ft. How many revolutions will it make in rolling one mile? (1 mi. = 5280 ft.)

13. A circular disk of clay is 20^{cm} in diameter. How many disks of the same thickness, 4^{cm} in diameter, can be made from the same clay? (L. LXXIX.)

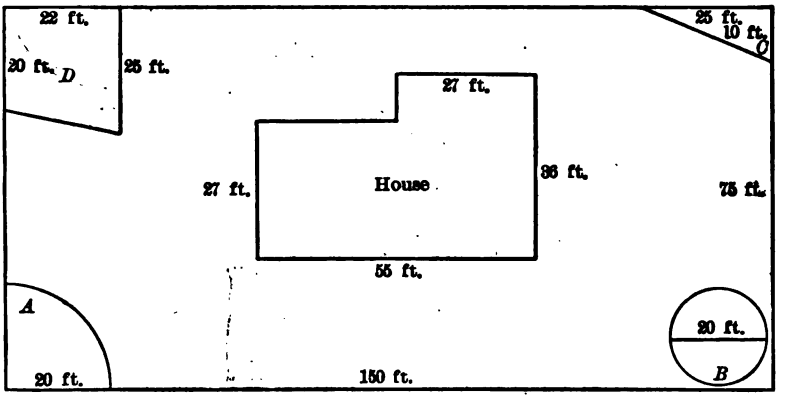
14. The diameter of a circle is 4 inches. Find the area of a sector of this circle, if the arc of the sector is 30°. (L. LXXIX.)

15. Find the total area of the walls, ceiling, and floor of a room 16 ft. long, 12 ft. wide, and 9 ft. high. (L. LXVIII.)

16. Find the entire surface of a triangular prism having an altitude of 20 in. and bases whose sides are 3 in., 4 in., and 5 in., respectively.

17. How many cubic centimeters in a cylinder whose altitude is 3 ft. and the diameter of whose bases is 1 ft.? (L. LXXXIX.)

18. How many square meters of surface on a cylinder 8^m long with ends 2^m in diameter? (L. LXXX.)



19. This figure represents a lot. A, C, and D are flower beds. B is a rock bed. Find the value of the lot at 5¢ per square foot.

20. Find the area of the flower bed represented by A.

21. How many more square feet are represented by D than by A?

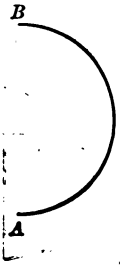
22. Which is larger, and how much, the flower bed C or the rock bed B?

23. How many square feet are covered by the house?

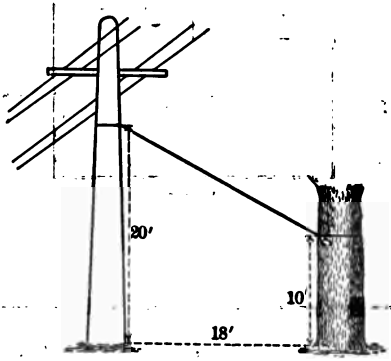
24. The rest of this lot is a lawn. How many square feet in the lawn?

NOTE. — The various results should be correct to two decimal places.

LXXXIV. PROBLEMS FOR THOUGHTFUL PUPILS

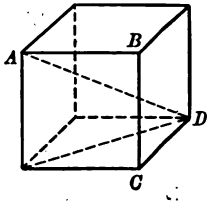


1. A man starts from A and travels 4 mi. an hour for 4 hours by way of the semicircle till he comes to B, directly opposite the center of the circle. How far is he from A?

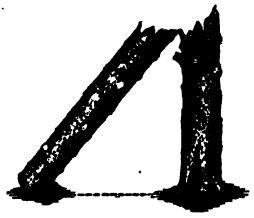


2. How long is the wire which connects the pole with the tree? (Not including the part wrapped about the tree and the pole.)

3. This drawing represents an 8-ft. cube. A fly walked from A to D, going along the edge AB, thence along the edge BC, thence along CD. How many feet was the trip? How far from A in a straight line is D?



4. A pole 63 ft. long broke off 28 ft. above the ground, the end broken off still resting upon the stump. How far is the end resting upon the ground from the foot of the stump?



5. A boy walked 9 miles north from his home, then 12 miles east. How many miles from home was he then?

NOTE. — If any result is not exact, give the answer to two decimal places.

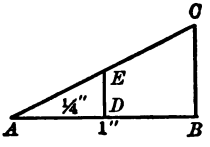


FIG. *a*.

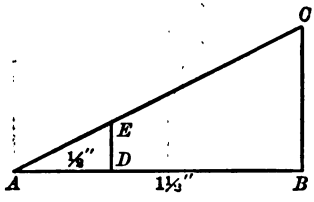


FIG. *b*.

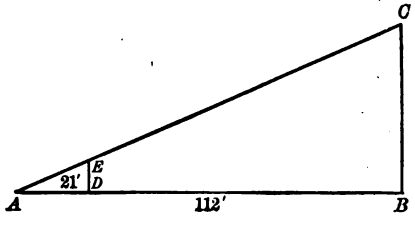
6. Fig. *a*. If AB equals twice AD, then BC must equal twice DE.

Fig. *b*. If AB equals 3 times AD, then BC must equal —— times DE.

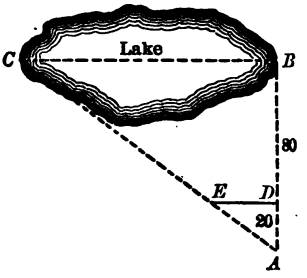
Make another right triangle in which AB equals 4 times AD.

Then BC must equal —— times DE. Verify your answer by measuring.

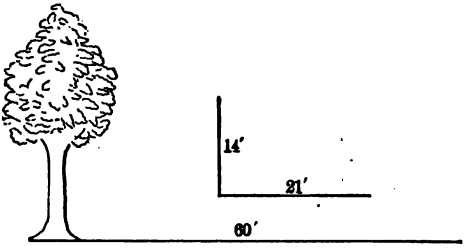
7. If AB is 112 ft. and DE is 21 ft. from A and 14 ft. high, how high is BC?



8. If AB is 80 rods, and AD is 20 rods, and ED is 25 rods, how long is the lake? The pupil should note the fact that CB and ED are perpendicular to AB in all of these examples.

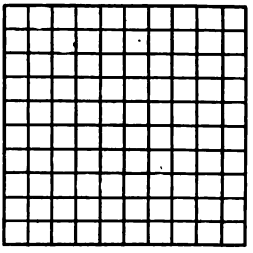


9. When a 5-foot stick casts a shadow 3 ft. long, how high is a church steeple that casts a shadow 60 ft. long?



10. A tree casts a shadow 60 ft. long at the same time that a pole 14 ft. long casts a shadow 21 ft. long. How high is the tree?

LXXXV. PERCENTAGE



1. This square is divided into how many equal squares?

2. Each small square is what fraction of the large square?

3. *Per cent* means *hundredths*. $\frac{1}{100}$ or .01 may be written 1 per cent or 1%. Each small square is 1% of the large square. 2 small squares are ——% of the large square. And so on.

4. Every business man must understand clearly the term *per cent.* He gains 10% if he gains \$10 on every \$100 he has paid for goods. The real estate agent charges $2\frac{1}{2}\%$ for purchasing a house and lot for a customer when he charges $\$2\frac{1}{2}$ for every \$100 paid for the property. A man lends money at 4% when he charges \$4 per year on every \$100 loaned. And so on.

5. A man borrowed \$560 for 1 year at 5%. How much interest did he pay?

5% = .05. .05 of \$560 = \$28 interest.

Or, interest at 5% means \$5 on each \$100 borrowed. \$560 = 5.60 hundred dollars. ∴ 5.6 × \$5, or \$28, is the interest required.

6. A dealer paid \$140 for a horse. He sold it at a gain of 12%. Find how much money he gained.

12% = .12. .12 of \$140 = \$16.80 gain.

Or, 12% gain means a gain of \$12 on each \$100 paid. \$140 = 1.40 hundred dollars. ∴ 1.4 × \$12, or \$16.80, is the gain.

LXXXVI. PERCENTAGE. FRACTIONS TO PER CENT

Express as hundredths and per cent; as,

$\frac{1}{2} = .50 = 50\%.$

$\frac{1}{3} = .33\frac{1}{3} = 33\frac{1}{3}\%.$

1. $\frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}.$

2. $\frac{2}{3}, \frac{3}{3}, \frac{1}{6}, \frac{5}{6}, \frac{1}{12}, \frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}.$

3. Out of 25 marbles 5 were lost. What part were lost? How many hundredths? What per cent?

4. A man bought a cow for $40 and sold her for $60. The gain was what fraction of the cost? How many hundredths? What per cent?

5. Blackberries were bought for 10¢ a quart and sold for 12¢ a quart. The gain is what fraction of the cost? How many hundredths? What per cent?

6. A horse was bought for $120 and sold for $80. The loss was what fraction of the cost? How many hundredths? What per cent?

7. What per cent profit is made by selling eggs for 45¢ a dozen if they cost 40¢ a dozen?

8. Out of 40 words given to Jane to spell she missed 8. What per cent of all did she miss?

9. A farmer had 600 bu. of corn one year and 800 bu. the next. What per cent was the increase? This means, what per cent of the first year's crop was the increase?

10. Cloth was bought at $3.50 per yard, and sold at $4.90 per yard. Find the gain per cent.

11. A sleigh was bought for $70, and sold for $56. How much was the gain per cent?

12. To find the gain or loss per cent when the cost price and the selling price are given, find the difference of the —— price and the —— price, then find what fraction that difference is of the —— price, and change the fraction to per cent.

LXXXVII. PERCENTAGE. PER CENT TO FRACTIONS

Express 20%, $37\frac{1}{2}\%$, and $62\frac{1}{2}\%$ as fractions in their lowest terms.

$$20\% = \frac{20}{100} = \frac{1}{5}.$$

$$37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{75}{200} = \frac{3}{8}.$$

$$62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}.$$

Express as fractions in their lowest terms:

1. 3%, 8%, 27%, $3\frac{1}{3}\%$, 5%, 4%, $8\frac{1}{3}\%$, $12\frac{1}{2}\%$.

2. $11\frac{1}{9}\%$, $14\frac{2}{7}\%$, $16\frac{2}{3}\%$, 20%, 25%, $33\frac{1}{3}\%$, $37\frac{1}{2}\%$, 40%.

3. $9\frac{1}{11}\%$, 100%, 125%, 150%, 225%, 275%.

4. A house which cost \$4000 was sold at a gain of 25%. The gain is equal to what fraction of the cost? How much was gained? Find the selling price.

5. Chickens were bought for $32¢$ each and sold at an advance of $37\frac{1}{2}\%$. Find the selling price.

6. Coffee was bought for $28¢$ a pound and sold at a loss of 25%. How much was the loss? What was the selling price?

7. Tea bought at $56¢$ a pound was sold at a gain of $14\frac{2}{7}\%$. Find the selling price.

8. A farmer who had 28 cows increased his herd $28\frac{4}{7}\%$. The increase was what fraction of the herd? How many cows had he after the increase?

9. To find the gain or loss when the cost price and the gain or loss per cent are given, multiply the —— price by the gain or loss per cent changed to a common or a decimal fraction.

To find the selling price, add or subtract the —— or the ——, as the case may be.

LXXXVIII. PERCENTAGE. RATES

Read the following decimals as per cents:

1. .10, .12½, .16⅔, .84, .05⅓, 1.24, 2.35.
2. .18, .19, .17½, .23½, .00½, .00¼, .00⅕.

Write as decimals:

3. 13%, 12½%, 27½%, .37½%, 125%, 235%.
4. ½%, ¾%, ⅕%, ⅕, ¼, ¼%.

Write as common fractions in their lowest terms:

5. ½%, ¼%, ⅕%, ⅖%, ⅗%, ⅘%, ⅛%.
6. ⅜%, $\frac{1}{10}$%, $\frac{3}{10}$%, ⅓%, ⅚%, ¾%, ⅞%.

Find 25% of $856.

25% = ¼. ¼ of $856 = $214.

Or, $856

.25

4280

1712

$214.00

We multiply by the rate expressed as a common fraction or as a decimal, using the form of the fraction which is most convenient.

7. Find 23% of 640 bu.

8. Of 3 tons of butter shipped to a neighboring city 7% was damaged. How many pounds were damaged?

9. How much will pay a bill of $420 after a reduction of 2%?

10. If $\frac{1}{8}$% is charged for buying stock, what is the commission for buying $3500 worth of stock?

11. 6% per year is what per cent per month?

12. $\frac{1}{3}$% a month is what per cent per year?

13. What is the cost of insuring the goods in a store valued at $32,500, the cost of the insurance being $\frac{1}{2}$% of the value of the goods?

LXXXIX. PERCENTAGE. THE PARTS

Find 7% of $640.

$640	base.	$640	multiplicand.
.07	rate.	.07	multiplier.
$44.80	percentage.	$44.80	product.

The principles of multiplications apply to percentage. The base is the multiplicand, the rate is the multiplier, and the percentage is the product.

The product of the base and the rate is the ——.

The quotient of the percentage divided by the rate is the ——.

1. Find 18% of 304.

304 is the ——, .18 is the ——. We are to find the ——.

2. 54.72 is 18% of what number?

54.72 is the ——, .18 is the ——. We are to find the ——.

3. To find the percentage when the base and rate are given, multiply the —— by the ——.

4. To find the base when the rate and the percentage are given, divide the —— by the ——.

5. 1075 is 25% more than what number?

$25\% = \frac{1}{4}$. Let x = the required number.

Then $x + \frac{1}{4}x = 1075$ the number + $\frac{1}{4}$ of it.

$\frac{5}{4}x = 1075$ 5 fourths of the number.

$\frac{1}{4}x = 215$ 1 fourth of the number.

$x = 860$ 4 fourths of the number.

XC. PERCENTAGE. TO FIND THE BASE

1. 360 is 4% of what number?

4)360	4% of the required number.
90	1% of the required number.
100	
9000	100% of the required number.

Or, 360 is the percentage or product, .04 is the rate or multiplier.

To find the base or multiplicand, divide the percentage or product by the rate or multiplier. .04)360.00 (L. LXXIX.)
9000

2. 164 is 8% of what number?

3. 11.9 is 7% of what number?

4. A man's interest payment at 4% a year was $168. On how much money was he paying interest?

5. My rent is $30 a month. In one year it equals 10% of the cost of the house. What was the cost of the house?

6. A man paid $720 for rent and groceries in one year. The rent was 8% and his groceries 12% of his salary. How much was his salary?

7. A lawyer charged $28 for collecting a debt. If his charge was 2% of the sum collected, how much was the debt?

8. I lent a friend some money. He pays me yearly $42 interest, the rate being 5%. How much did I lend him?

XCI. PERCENTAGE. AMOUNT AND DIFFERENCE

1. 55.2 is 20% more than what number?

20% = $\frac{1}{5}$. 20% more than —— = $1\frac{1}{5}$ of ——.

55.2 is $1\frac{1}{5}$, or $\frac{6}{5}$ of what number?

6)55.2 6 fifths of the required number.
9.2 1 fifth of the required number.
5
46.0 5 fifths of the required number.

Or, let $x =$ the required number.

Then $x + \frac{1}{5}x = 55.2.$

$\frac{6}{5}x = 55.2.$

$\frac{1}{5}x = 9.2.$

$x = 46.$ The required number.

2. 63 is 25% less than what number?

$25\% = \frac{1}{4}$. 25% less than the number $= \frac{3}{4}$ of the number.

63 is $\frac{3}{4}$ of what number?

3)63 3 fourths of the required number.

21 1 fourth of the required number.

4

84 4 fourths of the required number.

Or, let $x =$ the required number.

Then $x - \frac{1}{4}x = 63.$

$\frac{3}{4}x = 63.$

$\frac{1}{4}x = 21.$

$x = 84.$ The required number.

3. 112 is $33\frac{1}{3}\%$ more than what number?

4. 56 is $33\frac{1}{3}\%$ less than what number?

5. 140 is $16\frac{2}{3}\%$ more than what number?

6. 100 is $16\frac{2}{3}\%$ less than what number?

7. After an increase of 25% my flock of hens number 600. How many had I before the increase?

8. The population of a certain city in 1890 was 39,000, which was $33\frac{1}{3}\%$ less than the population in 1880. What was the population in 1880?

XCII. PERCENTAGE. MISCELLANEOUS

1. An agent collected $798, and charged 5% for his services. How much should he keep and how much should he pay over? What per cent is the amount paid over of the amount collected?

2. Express as decimals: 107%, 125%, 102%, 137½%.

Thus, $137\frac{1}{2}\% = \frac{137\frac{1}{2}}{100} = 1.37\frac{1}{2} = 1.375$.

3. A man bought a house and lot for $8500. 40% of this sum he paid in cash, and for the balance he gave a mortgage on the property. How much was the mortgage?

4. Sheep cost $6.75 each. At what price must they be sold to gain 33⅓%?

5. Some goods were bought for $250. An increase of 20% on this price was asked; but 5% being deducted from the asking price, for how much were the goods sold?

6. The population of a certain town is 25,000. What will be the population in 3 years, if it increases 3½% a year?

7. A merchant sold a customer some goods for $640 with the privilege of deducting 7% from the bill if he paid within 30 days. He paid in 10 days after the purchase; how much did the goods cost him? The sum deducted is called *trade discount.*

8. Sometimes successive discounts are made. How much money will pay a bill of $600 with 10, 20, and 5 off? This means, 10% of $600 from $600, then 20% of the remainder from the remainder, etc., each remainder serving as a new base.

9. *Copy and fill in the blanks:*

10% less than 20 = ——— % of 20.

3% more than 80 = ——— % of 80.

2% less than 50 = ——— % of 50.

4% more than 50 = ——— % of 50.

$2\frac{1}{2}$% more than 50 = ——— % of 50.

10. A merchant who made a profit of 12% gained $27 on a certain article. How much did he pay for it? (L. LXXX.)

11. A man paid $36 for goods. One half of them he sold at a gain of $11\frac{1}{9}$%. For how much must he sell the other half in order to gain 25% on the whole? (Total gain = 25% of ———.)

12. The cost price of a certain device was $2. The expense of selling it was 10% of the cost price. For how much must it be sold to gain 20% on the total cost?

13. A dealer paid $4140 for wheat, one third as much for rye, and three times as much for oats. He gained 8% on the wheat and 6% on the rye, but lost one half of 1% on the oats. How much was his total gain or loss?

14. A farmer paid $60 for one cow and $50 for another. He sold the first cow at a loss of 10% and the second cow at a gain of 15%. What was the selling price of each cow, and what was the gain or loss on the whole transaction?

15. A cubic foot of water weighs 1000 oz. Water is composed of 88.9% of oxygen and 11.1% of hydrogen. How many ounces of each are there in 10 cu. ft. of water? (88.9% = .889.)

16. The cost of an article is $84. It is sold at a gain of $28. What is the gain per cent?

17. Coffee bought at 24¢ a pound was sold at 28¢. Find the gain per cent.

18. A house which cost $3500 rented for $420 a year. For what per cent of its cost did it rent?

19. A grocer mixed 80 lb. of tea at 45¢ a pound with 100 lb. at 27¢ a pound and sold the mixture at 40¢ a pound. Find his per cent of profit.

20. A quantity of cloth was sold at an advance of $16\frac{2}{3}\%$. If the gain was $27, what was the cost? ($16\frac{2}{3}\%$ of the cost.)

21. If 5% more is gained by selling a bushel of grain for 96¢ than by selling it for 92¢, what is the original price? (5% of the cost.)

22. 40% of my money is invested in bank stock, and the rest, which is $36,000, is invested in mortgages. How much money have I? ($36,000 is what per cent of my money?)

23. A village property was sold for $4200, which was 20% more than it cost. Find the cost. (L. LXXXI.)

24. A dealer lost 7% by selling some grain for $799.80. Find the cost. (L. LXXXI.)

XCIII. PERCENTAGE. AGENCY

Much of the business in a country is done by persons acting for other persons as their agents.

The money paid to these agents for their services is called *commission* or *brokerage.*

An agent who buys and sells goods for another is called a *commission merchant.*

An agent who buys and sells stocks, bonds, promissory notes, etc., is called a *broker.*

In buying, the commission is charged on the *cost* price, and in selling it is charged on the *selling* price.

(*a*) A hay dealer sold 30 tons of hay at \$15 a ton, and received a commission of 12%. How much was his commission? How much did he send to his employer?

\$15 × 30 = \$450, selling price.

12% of \$450 = .12 of \$450 = \$54, commission.

∴ \$450 − \$54 = \$396, amount sent to employer.

(*b*) \$35.52 commission was charged for selling \$960 worth of goods. Find the rate of commission.

The commission = $\frac{35.52}{960}$ of the selling price = .037.

∴ the rate of commission is 3.7%.

(*c*) A commission merchant's commission at $2\frac{1}{2}$% for buying some fruit was \$75. What was the cost of the fruit?

$2\frac{1}{2}$% of the cost = \$75.

1% of the cost = \$30.

100% of the cost = \$3000, the cost.

Or, .025 × (\$) = \$75.

∴ \$75 ÷ .025 = \$3000, the cost.

Refer to (*a*), (*b*), and (*c*).

1. A commission merchant sold 560 bbl. of apples at $3.25 a barrel on a commission of 2%. Find his commission. Find the amount remitted to his employer. This amount is what per cent of the selling price? See (*a*).

2. If I sell 240 bbl. of lime at $1.50 a barrel on a commission of $1\frac{1}{2}\%$, how much is my commission? How much should I send to my employer? What I send to my employer is what per cent of the selling price?

3. For buying $8462 worth of goods a commission of $253.86 was charged. Find the rate of commission. See (*b*).

4. A commission of $264.74 was charged for buying $7564 worth of goods. What was the rate of commission?

5. An agent who charges $3\frac{1}{2}\%$ commission for selling goods received $25.90 as his commission on a certain sale. What was the selling price of the goods? See (*c*).

6. An agent received $32.41 as his commission on a certain purchase. If his rate of commission was $2\frac{1}{2}\%$, what was the purchase price?

XCIV. PERCENTAGE. INSURANCE

1. Fire insurance is a contract to guarantee security against loss of property by fire.

2. There is also insurance against accident, robbery, and death.

3. The sum of money paid to the insurance company for the insurance is the premium.

4. The written contract is the policy.

(*a*) My house was insured for $3000 at the rate of $\frac{1}{4}\%$ for one year. How much was the premium?

$\frac{1}{4}\%$ of $3000 = .00\frac{1}{4}$ of $3000 = $7.50, premium.

Or, $\frac{1}{\cancel{400}\ 2}$ of $\frac{\overset{15}{\cancel{3000}}}{1} = \frac{15}{2} = $7.50, premium.

(*b*) A premium of $360 was charged for effecting an insurance of $24,000 on a house. Find the rate per cent of premium.

The premium $= \frac{360}{24000}$ of the insurance $= .015 = 1\frac{1}{2}\%$, rate.

5. A factory was insured for $28,000 at the rate of $\frac{3}{8}\%$ for one year. Find the amount of the premium. See (*a*).

6. A shipment of goods was insured for $8600, paying premium at the rate of $\frac{3}{5}\%$. How much did the premium amount to?

7. A premium of $30 was paid for an insurance of $4500. What was the rate of premium? See (*b*).

8. $21.60 premium was paid to secure an insurance of $5400. What was the rate of premium?

XCV. PERCENTAGE. TAXES AND DUTIES

1. Money assessed on persons or property for public purposes is called a *tax*.

2. Direct taxes are levied by the state, county, township, city, or the school district.

3. Indirect taxes are called *duties*, and are levied by the United States on imported goods.

4. Duties reckoned at a certain rate per cent on the cost of the goods in the country where they were bought are called *ad valorem* duties. *Ad valorem* means "according to value."

5. Duties reckoned at a fixed charge per pound, per gallon, per yard, etc., regardless of the cost of the goods, are called *specific* duties.

(*a*) The taxable valuation of the property of a certain town is $2,400,000, and a tax of $4800 is to be levied for furnishing a schoolhouse. What will be the rate of taxation? What will be the tax paid by John's father if his property is assessed at a valuation of $8960?

The tax $= \frac{4800}{2400000}$ of the valuation $= .002$ (or 2 mills on a dollar). The tax of every taxpayer will be .002 of his assessed valuation.

∴ John's father will pay .002 of $8960 = $17.92.

6. How much tax will Henry's father have to pay, his assessed valuation being $12,970?

7. Mary's father is assessed for $21,000. How much will his tax be?

8. A county levied a tax of $2175 for building a bridge. The taxable valuation being $1,450,000, what is the tax rate? How much was A's tax, whose taxable valuation was $5700?

9. How much was B's tax, if his assessed valuation was $9600?

10. A certain school district, with an assessed valuation of $1,250,000, built a schoolhouse for $6875. What was the tax rate? How much was A's tax, if his valuation was $12,600?

11. How much was B's tax, his valuation being $8450?

12. What is the duty on an importation of 840 yd. of silk bought at 7 francs a yard, the duty being 25%? (1 franc = 19.3¢.)

13. In Ex. 12 the duty is ad valorem. What is the specific duty on 890 gal. of alcohol at 15¢ a gallon?

14. If the ad valorem duty on silk at $37\frac{1}{2}\%$ was $600, what was the invoice price of the silk? (The cost price.)

NOTE. — Taxes and duties should be more fully treated in connection with civil government.

XCVI. PERCENTAGE. INTEREST

The sum paid for the use of a house is called *rent.*

The sum paid for a trip on the cars is called *fare.*

The sum paid for the services of a man is called *wages* or *salary.*

The sum paid for the use of money is called *interest.*

The money borrowed is called the *principal.*

The principal and the interest added together become the *amount.*

The *rate of interest* is a certain rate per cent of the principal for 1 yr.

1. If I borrow $700 for 1 yr., what must I pay for the use of it if the rate of interest is 6%?

$700 principal.
.06 rate.
$42.00 interest.

At the beginning of the year I owe $700, at the end of the year I owe $——, the amount.

2. Find the amount of $874.67 for 2 yr. 6 mo. at 7%.

$874.67 principal.
.07 rate.
$61.2269 interest for 1 yr.

2 yr. 6 mo. = $2\frac{1}{2}$ yr.

$61.23 interest for 1 yr.
$2\frac{1}{2}$
$3061\frac{1}{2}$
12246
153.07\frac{1}{2}$

$153.08
874.67
$1027.75 amount.

The interest for 1 yr. = 7% of $874.67 = $61.23.

The interest for 2 yr. 6 mo. = $2\frac{1}{2}$ × $61.23 = $153.08.

The amount = $874.67 + $153.08 = $1027.75.

3. Find the amount of $600 from April 14, 1900, to Oct. 18, 1900, at 5%.

From April 14 to Oct. 14 is 6 mo.
From Oct. 14 to Oct. 18 is 4 da.

The time is

$$6 \text{ mo. } 4 \text{ da.} = 6\tfrac{4}{30} \text{ mo.} = 6\tfrac{2}{15} \text{ mo.} = \frac{6\frac{2}{15}}{12} \text{ yr.} = \frac{92}{15} \times \frac{1}{12} = \frac{23}{45} \text{ yr.}$$

$$\frac{\$600}{1} \times \frac{5}{100} \times \frac{23}{45} = \frac{46}{3} = \$15.33, \text{ interest.}$$

$$\$600 + \$15.33 = \$615.33, \text{ amount.}$$

Find the interest on:

4. $85 for 1 yr. at 4%.
5. $720 for 1 yr. at 5%.
6. $946 for 1 yr. at 6%.
7. $85 for 2 yr. at 4%.
8. $720 for 3 yr. at 5%.
9. $946 for 4 yr. at 6%.
10. $85 for 6 mo. at 4%.
11. $720 for 8 mo. at 5%.
12. $946 for 9 mo. at 6%.
13. $85 for 120 da. at 4%.
14. $720 for 60 da. at 5%.
15. $946 for 30 da. at 6%.

Find the amount of:

16. $860 for 2 yr. 6 mo. at 6%.
17. $1060 for 5 yr. 8 mo. at 7%.
18. $720 for 3 yr. 4 mo. at 8%.

19. $3090 for 4 yr. 3 mo. at 5%.

20. $720 for 2 yr. 3 mo. at 4%.

21. $4080 for 1 yr. 10 mo. at $4\frac{1}{2}$%.

22. $600 from May 20, 1900, to Aug. 30, 1900, at 7%.

23. $400 from June 27, 1903, to Oct. 27, 1903, at 5%.

XCVII. PERCENTAGE. INTEREST

SIX PER CENT METHOD

At 6%:

The interest for 1 yr. $= .06$ of the principal.

The interest for 1 mo. $= \frac{1}{12}$ of $.06 = .005$ of the principal.

The interest for 1 da. $= \frac{1}{30}$ of $.005 = .000\frac{1}{6}$ of the principal.

Find the interest of $620 for 2 yr. 7 mo. 15 da. at 6%.

At 6% the interest for:

2 yr. $= 2 \times .06 = .12$ of the principal.
7 mo. $= 7 \times .005 = .035$ of the principal.
15 da. $= 15 \times .000\frac{1}{6} = .0025$ of the principal.

2 yr. 7 mo. 15 da. $= .1575$ of the principal.

Therefore the interest $= .1575 \times \$620 = \$96.95$.

NOTE. — The interest at 7% may be found by increasing the interest at 6% by $\frac{1}{6}$ of itself. The interest at 5% may be found by diminishing the interest at 6% by $\frac{1}{6}$ of itself.

1. The interest at 8% = the interest at 6% plus ——— of itself.

2. The interest at 9% = the interest at 6% plus ——— of itself.

3. The interest at $7\frac{1}{2}\%$ = the interest at 6% plus ——— of itself.

4. The interest at 5% = the interest at 6% less ——— of itself.

5. The interest at 4% = the interest at 6% less ——— of itself.

6. The interest at $4\frac{1}{2}\%$ = the interest at 6% less ——— of itself.

Find the interest on:

7. $840 for 2 yr. 6 mo. 12 da. at 6%.

8. $86.50 for 5 yr. 8 mo. 18 da. at 6%.

9. $104.25 for 3 yr. 2 mo. 15 da. at 6%.

10. $98.40 for 1 yr. 5 mo. 3 da. at 7%.

11. $900 for 5 yr. 1 mo. 20 da. at 8%.

12. $84.75 for 4 yr. 5 mo. 21 da. at 4%.

XCVIII. PERCENTAGE. INTEREST

SIX PER CENT METHOD BY DAYS

At 6% the interest for 1 yr. equals $\frac{6}{100}$ of the principal, and the interest for 1 day equals $\frac{1}{360}$ of $\frac{6}{100}$, or $\frac{1}{6000}$ of the principal. Therefore, the interest for 1 day, at 6%,

may be found by dividing the principal by 6000. This may be done by moving the decimal point three places to the left and dividing by 6.

1. Find the interest on $648 for 35 days at 6%.

6)$.648 after moving the decimal point 3 places to the left.
$.108 the interest for 1 day.
35
540
324
$3.780 the interest for 35 days.

Find the interest on:

2. $870 for 45 days at 6%.
3. $4060 for 60 days at 6%.
4. $860 for 72 days at 6%.
5. $90.60 for 90 days at 6%.
6. $720 from May 10, 1896, to June 9, 1896, at 6%.
7. $96 from April 15, 1903, to May 13, 1903, at 6%.
8. $1094.70 from July 10, 1904, to Aug. 1, 1904, at 6%.
9. $1124 from Aug. 14, 1904, to Sept. 17, 1904, at 6%.
10. $500 for 96 days at 7%. (Note in L. LXXXVI.)
11. $8060 for 120 days at 5%. (Note in L. LXXXVI.)
12. $75 for 45 days at 4%.
13. $900 for 75 days at $4\frac{1}{2}$%.

XCIX. PERCENTAGE. EXACT INTEREST

The United States government computes interest for periods less than a year by taking that fraction of a year's interest that the exact number of days is of 365 days. The interest on $500 for 5 da. at 4% would be found thus:

$$\frac{\$\,\overset{5}{\cancel{500}}}{1} \times \frac{4}{\underset{1}{\cancel{100}}} \times \frac{5}{\underset{73}{\cancel{365}}} = \$\frac{20}{73} = \$.27. \quad \textit{Ans.}$$

Find the exact interest on:

1. $750 for 120 da. at 5%.
2. $800 for 60 da. at 4½%.
3. $6900 for 80 da. at 4%.
4. $5200 for 90 da. at 4½%.
5. $3200 from Jan. 1, 1900, to July 16, 1900, at 3½%.
6. $7000 from Feb. 2, 1900, to Aug. 15, 1900, at 4%.
7. $9000 from March 1, 1901, to Nov. 1, 1901, at 4½%.
8. $5760 from June 1, 1903, to Nov. 15, 1903, at 3½%.
9. $6900 from May 7, 1902, to July 16, 1903, at 4½%.

From May 7, 1902, to May 7, 1903, is 1 yr.
From May 7, 1903, to June 16, 1903, is 70 da.

$$1 \text{ yr. } 70 \text{ da.} = 1\tfrac{70}{365} \text{ yr.} = 1\tfrac{14}{73} \text{ yr.}$$

$$\$\tfrac{6900}{1} \times .04\tfrac{1}{2} \times 1\tfrac{14}{73} = \$\tfrac{6900}{1} \times \tfrac{9}{200} \times \tfrac{87}{73} = \$370.05. \quad \textit{Ans.}$$

Find the exact interest on:

10. $8000 from Jan. 16, 1903, to Feb. 19, 1904, at 4%.

11. $8960 from July 1, 1902, to Jan. 15, 1904.

12. $12,000 from April 1, 1901, to June 6, 1902.

C. PERCENTAGE. INTEREST. REVIEW

1. Find the interest on $875 for 3 yr. 4 mo. 6 da. at 6%.

2. Find the amount of $9000 for 4 yr. 2 mo. 12 da. at 6%.

3. I borrowed $1060, and after 1 yr. 6 mo. I returned it with interest at 6%. How much did I pay?

4. I have $5000 U.S. bonds which yield me $4\frac{1}{4}\%$ interest. How much is my interest from Jan. 1, 1904, to Sept. 4, 1904? (L. LXXXVIII.)

5. A friend lent me $2000. How much should I pay him after keeping the money 2 yr. 8 mo. 12 da., interest being at the rate of 6%?

6. How many years would it take for the interest to equal the principal, if the rate is 6%?

7. In how many years will money double itself at 4%?

8. I borrowed $600 on April 1, 1903, at 6%. How much must I pay on Nov. 13, 1903, to cancel the debt?

9. At 5%, what is the interest on $3500 for 1 yr.? In how many years would the interest become $787.50?

10. What is the exact interest on $3800 for 73 days at 5%?

11. At 4%, $600 would gain how much interest in 1 yr.? How long would it take the same money to gain $6 interest?

12. If the interest on $6000 in one year is $270, what is the rate per cent?

CI. PERCENTAGE. INTEREST. NOTES

Mr. Jones wanted to buy a horse owned by Mr. Dayton, but he was unable at once to pay the $200 which Mr. Dayton asked for the animal. Mr. Dayton said that he would accept a note for the money from Mr. Jones, whereupon Mr. Jones bought the horse and gave Mr. Dayton the following note:

200\frac{00}{100}$. ALBANY, N.Y., June 6, 1902.

Sixty days after date, I promise to pay to the order of R. E. Dayton Two Hundred Dollars, value received.

J. D. JONES.

The *face* of this note is $200.
The *date* is June 6, 1902.
The *time to run* is 60 da.
The *payee* is R. E. Dayton.
The *maker* is J. D. Jones.

The note becomes due 60 da. after June 6, 1902, which is Aug. 5, 1902, when Mr. Jones should pay Mr. Dayton the $200.

The *date of maturity* is Aug. 5, 1902.

If Mr. Jones fails to pay the note till, say, Sept. 5, 1902, he must pay interest on the $200 from the date of maturity till Sept. 5, 1902.

This is an example of a note without interest, no interest being required, till after the note becomes due.

Study this note and write one of the same form for $600 for 90 days.

$960\frac{00}{100}$. DOVER, N.J., July 16, 1897.

Ninety days after date, I promise to pay to the order of J. D. Kimberly Nine Hundred Sixty Dollars, at the First National Bank, value received. M. J. MABIE.

1. What is the *face* of this note?
2. What is the *time to run?*
3. Who is the *payee?*
4. Who is the *maker?*
5. What is the *date of maturity?*
6. Where is the note payable?

This note is called a *bank note*, because it is made payable at a bank.

7. $680\frac{00}{100}$. HARRISBURG, PA., Oct. 3, 1899.

Sixty days after date, I promise to pay to the order of A. B. Mosher Six Hundred Eighty Dollars, with interest, at the Second National Bank, value received.

R. M. MORRIS.

Find the value of this note at maturity, the rate of interest being 6%.

8. $\$800\frac{00}{100}$. POUGHKEEPSIE, N.Y., May 4, 1903.

Four months after date, I promise to pay to the order of James Zabriskie Eight Hundred Dollars, with interest, at the First National Bank, value received.

JOHN MINER.

Find the value of this note at maturity, the rate of interest being 6%.

9. What sum will pay at maturity a 90-day note for $\$750$ with interest at 5%?

10. Write a bank note for $\$750$ for 60 days.

CII. PERCENTAGE. PARTIAL PAYMENTS

When payments on notes are made from time to time, each payment and its date are written on the back of the note as a receipt. Such acknowledgments are called *indorsements*.

Pupils should work this example as well as those that follow.

1. $\$700\frac{00}{100}$. NEW YORK, N.Y., March 1, 1901.

One month after date, I promise to pay to the order of Stephen Smith Seven Hundred Dollars, with interest, for value received.

EUGENE BIGELOW.

On this note were the following indorsements: Aug. 1, 1901, $\$40$; Jan. 2, 1902, $\$15$; March 2, 1902, $\$200$; May 30, 1902, $\$300$. How much was due April 16, 1903, interest at 6%?

Face	$700.00
Interest on $700 from March 1, 1901, to Aug. 1, 1901 (5 mo.)	17.50
Amount due at time of first payment	717.50
First payment to be deducted	40.00
First new principal	677.50
Interest on $677.50 from Aug. 1, 1901, to Jan. 2, 1902 (5 mo. 1 da.)	17.05
The 2d payment, being less than the interest due, is not deducted.	
Interest on $677.50 from Jan. 2, 1902, to March 2, 1902 (2 mo.)	6.78
Amount due March 2, 1902	701.33
Sum of 2d and 3d payments to be deducted	215.00
Second new principal	486.33
Interest on $486.33 from March 2, 1902, to April 16, 1903 (1 yr. 1 mo. 14 da.)	32.75
Amount due April 16, 1903	$519.08

When partial payments are made on interest-bearing debts running longer than one year, the above method is authorized by the United States courts, and it is called the *United States Rule.* It is not lawful for either the payment or the interest to bear interest. Notice that the 2d payment ($15) is less than the interest ($17.05) due, and is not subtracted as in the case of the 1st payment. If the $17.05 were added to the 1st new principal ($677.50), and the 2d payment deducted, there would result a new principal, $679.55. If now we should compute the interest on $679.55, we should be computing *interest on interest*, and would thus violate the rule.

When a payment is less than the interest due, we neglect it until such time as the payments taken together are equal to, or greater than, the interest due. We then proceed as with the first payment. The sum of the 2d and 3d payments ($215) being not less than the interest due ($23.83), we add the interest, $17.05 and $6.78, to $677.50, and subtract the sum of the two payments, $215.

2. 5000\frac{00}{100}$. NEW YORK, N.Y., Aug. 10, 1903.

Two months after date, I promise to pay to the order of James Dougal Five Thousand Dollars, with interest at 6%, for value received. R. C. SHAFER.

This note bore the following indorsements: Jan. 15, 1904, $300; July 27, 1905, $1750. How much was due Aug. 31, 1905?

3. 6000\frac{00}{100}$. NEWARK, N.J., Feb. 15, 1893.

Six months after date, I promise to pay to the order of Henry Blair Six Thousand Dollars, with interest at 6%, for value received. JAMES HENDRICKS.

On this note were indorsed the following payments: Aug. 1, 1893, $140; Feb. 21, 1894, $675; July 27, 1894, $1500. Find the amount due Dec. 27, 1894.

4. 3000\frac{00}{100}$. MONTPELIER, VT., March 10, 1900.

Sixty days after date, I promise to pay to James Madison, or order, Three Thousand Dollars, with interest at 6%, for value received. JOHNSON BRIDGMAN.

The following indorsements were on this note: March 22, 1901, $1750; June 16, 1901, $900. Find the amount due March 4, 1902.

5. $\$600\frac{00}{100}$. SPRINGFIELD, MASS., May 9, 1898.

Thirty days after date, I promise to pay to the order of Elmer Stanford Six Hundred Dollars, with interest at 6%, for value received. JOHN RAYMOND.

This note bore these indorsements: May 9, 1899, $400; May 9, 1900, $200. How much was due May 9, 1901?

CIII. NOTES. CHECKS

$\$$_____ *Newark, N. J.,* _____ *19*_____

_____ *after date* _____ *promise to pay*

to the order of _____

_____ $\frac{}{100}$ *Dollars*

at _____ *for value received.*

*No.*_____ *Due*_____ _____

A bank note. A printed form ready for use.

*No.*_____ *New York,* _____ *190*_____

Riverside Bank

8th AVE. COR. 57th ST.

Pay to _____ *or Order*

_____ *Dollars.*

$\$$_____ _____

A bank check. A printed form ready for use.

On the preceding page are given a blank bank note and a blank bank check ready for filling in. The entire note or check may be in writing, but the printed forms in general use save much time in practice.

1. Make a copy of the above note in writing, making a note for $900 due 90 days after date, and supplying all data omitted.

2. Make a copy of the above check in writing, making the check for $50, and supplying all data omitted.

A book containing blank checks is given by the bank to each depositor for his use in drawing on his deposits to pay his expenses or for other uses.

3. Suppose the check just given to be filled out thus:

No. 672. *New York, Jan. 17, 1905.*

Riverside Bank

8th AVE. COR. 57th ST.

Pay to J. R. Staley ..... *or Order*

Seven and $\frac{50}{100}$ ..... *Dollars.*

$7\frac{50}{100}$. *W. J. Waverly.*

J. R. Staley can obtain seven dollars and fifty cents on this check by writing his name across the back of the check and presenting it at the Riverside Bank. He is said to indorse the check when he thus writes his name on the back. He may not present it at the bank, but after indorsing it he may use it as money to the extent of the amount named.

It would be well for him to write above his indorsement the words "Pay to the order of —— ——," writing the name of the person to whom he gives the check; otherwise any person finding the check with his name alone on the back might be able to collect the money. It is much more difficult for a dishonest person finding a check indorsed to some particular person to collect the money than if the check were indorsed "in blank," as we say when only the payee's name appears on the back.

CIV. PERCENTAGE. BANK DISCOUNT

1. $600. NEW YORK, N.Y., Jan. 6, 1897.

Sixty days after date, I promise to pay to the order of James Doty Six Hundred Dollars, for value received.

JOHN HENRY.

If James Doty wishes to get some money from a bank on this note, he will write his name on the back of the note and present it to the bank. If either man is considered responsible for the money, the bank will pay James Doty not the $600 in full, but $600 less the interest on $600 for 60 days.

The interest on $600 for 60 da. at 6% is $6. The bank will pay James Doty $600 − $6 = $594. When the note becomes due, John Henry will pay the bank $600.

The *bank discount* is $6. The *proceeds* is $594. If James Doty had waited till the note became due, he would have received $600; but he preferred, for some reason, $594 at once to $600 sixty days after Jan. 6, 1897.

In this case the *term of discount* is 60 days.

In some states three days, called "days of grace," in addition to the time named in the note, are included in the "time to run," and the note in such a case is not legally due till the expiration of the days of grace.

In this book it will be understood that there are no "days of grace," unless they are specified.

2. $400. DETROIT, MICH., May 3, 1903.

Three months after date, I promise to pay to the order of Simon Johnson Four Hundred Dollars, with interest at 6%, for value received. JAMES SMALLEY.

As 3 days of grace are allowed in Michigan, the time to run is 3 calendar months plus 3 days, making the date of maturity Aug. 6, 1903.

If the note had read "ninety days after date," the date of maturity would have been 93 days after May 3, 1903, or Aug. 4, 1903. In general, if the time in the note is expressed in months, calendar months are reckoned in finding the date of maturity. If the time is expressed in days, the *exact* number of days is reckoned from the date of the note.

The interest at 6% for

$$\begin{aligned} 3 \text{ mo.} &= .015 \text{ of the principal.} \\ 3 \text{ da.} &= .0005 \text{ of the principal.} \\ 3 \text{ mo. } 3 \text{ da.} &= .0155 \text{ of the principal.} \end{aligned}$$

$.0155 of $400 = $6.20.

$400 + $6.20 = $406.20 value at maturity.

This note was discounted May 15, 1903. (*a*) Find the discount. (*b*) Find the proceeds.

The *term of discount* is the exact number of days from May 15, 1903, to Aug. 6, 1903, which is 83 days, found thus: beginning May 15, the rest of

May = 16 days.
June = 30 days.
July = 31 days.
Aug. = 6 days.

May 15 to Aug. 6 = 83 days.

The sum to be discounted is always the value at maturity. Therefore, $406.20 is to be discounted for 83 days.

6)$.40620
.0677
83
2031
5416
$5.6191

(*a*) $5.62 the discount.

$406.20
5.62

(*b*) $400.58 the proceeds.

Find the date of maturity, term of discount, discount, and proceeds.

3. $700. ROCHESTER, N.Y., Oct. 28, 1899.

Four months after date, I promise to pay to the order of Miles Standish Seven Hundred Dollars, for value received.

MICHAEL SCHEILER.

This note was discounted Oct. 28, 1899. Rate, 6%.

4. $850. BOSTON, MASS., Jan. 27, 1900.

Ninety days after date, I promise to pay to the order of James Krueger Eight Hundred Fifty Dollars, value received.

ALEXANDER MARSH.

This note was discounted Feb. 16, 1900. Rate, 6%.

5. $500. NEWPORT, R.I., March 26, 1895.

Thirty days after date, I promise to pay to the order of Jonathan Edwards Five Hundred Dollars, value received.

JOSIAH QUINCY.

This note was discounted April 13, 1898. Days of grace. Rate, 6%.

6. $3000. BATON ROUGE, LA., May 2, 1899.

Two months after date, I promise to pay Julius Scribner or order Three Thousand Dollars, with interest at 6%, value received.

MARY JONES.

This note was discounted June 3, 1899. Days of grace. Rate, 6%. See **2**.

7. $200. SYRACUSE, N.Y., June 17, 1903.

Ninety days after date, I promise to pay to the order of R. M. Major Two Hundred Dollars, for value received.

C. J. MINOR.

This note was discounted at date of issue. Rate, 6%.

8. $875. BUFFALO, N.Y., Nov. 1, 1903.

Ninety days after date, I promise to pay to the order of Timothy Lamont Eight Hundred Seventy-five Dollars, value received.

AMOS JENNINGS.

This note was discounted on the day of issue. Rate, 6%.

9. A 90-day note for $700 was discounted 30 days after it was issued. Find the proceeds. Rate, 6%.

10. A 90-day note for $700, with interest, was discounted 30 days after it was issued. Find the proceeds. Rate, 6%.

11. A 60-day note for $250 was discounted 30 days after date. Find the proceeds. Rate, 6%.

12. A 60-day note for $250, with interest, was discounted 30 days after date. Find the proceeds. Rate, 6%.

CV. PERCENTAGE. COMPOUND INTEREST

If I owe $100 at 6% interest for one year, at the end of the year I owe $106. If I then fail to pay the money and do not pay the interest, I should pay interest for the second year on $106. The interest on $106 for 1 year is $6.36. Then I would owe $106 plus $6.36, or $112.36. If I still pay nothing, I should pay interest for the third year on $112.36, and so on until I pay. This method of adding the interest to the principal at regular intervals and thus forming new principals, is called *compounding interest.*

In *compound interest* the interest is computed on both principal and interest, while in *simple interest* the interest is computed upon the principal alone.

This is the method used in savings banks when the interest is not drawn out by the depositor at the close of each interest period. The interest periods in many savings banks are 6 months.

1. What is the compound interest of $400 for 4 yr. at 5%?

$400	principal.
.05	
20.00	interest at the end of the 1st year.
400	
$420	2d principal. Amount at the end of
.05	the 1st year.
21.00	interest at the end of the 2d year.
420	
$441	3d principal. Amount at the end of
.05	the 2d year.
22.05	interest at the end of the 3d year.
441	
$463.05	4th principal. Amount at the end of
.05	the 3d year.
23.1525	interest at the end of the 4th year.
463.05	
$486.2025	5th principal. Amount at the end of
400	the 4th year.
$86.2025	compound interest.

The pupil will note that $486.20 is the *amount* of $400 at compound interest for the 4 yr. As the *amount* includes both the principal and the interest, it is evident that the compound *interest* may be found by subtracting the principal, $400, from the amount, $486.20. This gives the compound interest, $86.20.

The example may be done as follows:

\$400	
1.05	
2000	
400	
\$420	amount at the end of the 1st year.
1.05	
2100	
420	
\$441.00	amount at the end of the 2d year.
1.05	
2205	
441	
\$463.05	amount at the end of the 3d year.
1.05	
231525	
46305	
\$486.2025	amount at the end of the 4th year.
400	
\$86.2025	compound interest.

2. Find the compound interest of \$450 for 2 yr. at 5%.

3. Find the compound interest of \$975 for 3 yr. at 4%.

4. Find the compound interest of \$1075 for 4 yr. at 4%.

5. Find the compound interest of \$5000 for 4 yr. at $3\frac{1}{2}$%.

6. Find the compound interest of \$600 for 1 yr. 6 mo. at 4%, compounded semiannually. Notice that the periods are a half year each, and that 1 yr. 6 mo. make three periods. The rate is one half of 4%, or 2%, for each period.

7. Find the compound interest of $750 for 2 yr. at 4%, compounded semiannually.

8. Find the compound interest of $1500 for 1 yr. 9 mo. at 4%, compounded quarterly. 1% each period.

9. Find the compound interest of $1000 for 1 yr. 3 mo. at 6%, compounded quarterly.

10. Find the compound interest of $1200 for 2 yr. 4 mo. 6 da. at 6%. Use the amount at the end of the 2 yr. as the principal upon which to find the amount for the remaining time.

11. Find the compound interest of $1100 for 2 yr. 8 mo. 6 da. at 6%.

12. Find the compound interest of $2000 for 3 yr. 4 mo. 12 da. at 4%.

CVI. LONGITUDE AND TIME (*a*)

We have learned that every circumference is divided into 360 equal parts, called *degrees*.

Now we must learn that every degree is divided into 60 equal parts, called *minutes*, and each minute is divided into 60 equal parts, called *seconds*.

Thus we have the table of

CIRCULAR (OR ANGULAR) MEASURE

60 seconds (″) = 1 minute (′).

60 minutes = 1 degree (°).

360 degrees = 1 circumference (C.).

The imaginary lines on the earth's surface running north and south, and represented on the maps in geography, are called *meridians*. The *prime* meridian is the one which passes through Greenwich, near London. Places west of the *prime* meridian are in west longitude, and places east of the *prime* meridian are in east longitude. New York is 74° 3″ west longitude. Paris is 2° 20′ 9″ east longitude.

1. Washington is 77° 0′ 28″ W. longitude, and Boston is 71° 3′ 58″ W. longitude. What is the difference in longitude?

2. The longitude of Chicago is 87° 34′ 9″ W., and that of New York is 70° 0′ 3″ E. What is the difference in longitude?

3. Vienna is 16° 23′ E. longitude, and Marseilles is 5° 22′ E. longitude. What is the difference in longitude?

4. The longitude of St. Paul, Minn., is 93° 5′ W., and that of St. Petersburg is 30° 17′ 56″ E. What is the difference in longitude?

CVII. LONGITUDE AND TIME (*b*)

As the earth turns around from west to east once in 24 hours, the sun appears to make one circumference in its apparent path around the earth from east to west in 24 hours.

The sun appears to travel in 24 hours through a circumference, or 360° of longitude; it travels in 1 hr. through $\frac{1}{24}$ of 360°, or 15°; in 1 min. it travels through $\frac{1}{60}$ of $15° = \frac{1}{4}° = 15'$; in 1 sec. it travels through $\frac{1}{60}$ of $15' = \frac{1}{4}' = 15''$.

Thus we obtain the following:

15° of longitude correspond to 1 hour of time.
15′ of longitude correspond to 1 minute of time.
15″ of longitude correspond to 1 second of time.

Notice that the number of degrees, minutes, and seconds of *longitude* is 15 times the corresponding number of hours, minutes, and seconds of *time;* also, that the number of hours, minutes, and seconds of *time* is $\frac{1}{15}$ of the number of degrees, minutes, and seconds of *longitude.*

For convenience the United States is divided by certain meridians into four sections, 15° of longitude each. These are named according to location the Eastern, Central, Mountain, and Pacific sections. The meridians locating these sections are the 75th, 90th, 105th, and 120th west of the meridian which passes through Greenwich, England, which is usually regarded as the prime meridian for the world.

The true time of the 75th meridian west of Greenwich is 5 hours slower than Greenwich time. All places within $7\frac{1}{2}$° on either side of the 75th meridian are in the Eastern section and have the clock time of that meridian. These places have Eastern standard time.

The true time of the 90th meridian west of Greenwich is 6 hours slower than Greenwich time. All places within $7\frac{1}{2}$° on either side of the 90th meridian are in the Central section and have the clock time of that meridian. These places have Central standard time.

1. How many hours slower than Greenwich time is the true time of the 105th meridian west of Greenwich? (Read carefully all that precedes, and refer to a map.)

2. What name is given to the section $7\frac{1}{2}°$ on either side of the 105th meridian west of Greenwich?

3. How many hours slower than Greenwich time is the true time of the 120th meridian west of Greenwich?

4. What name is given to the section $7\frac{1}{2}°$ on either side of the 120th meridian west of Greenwich?

5. How many hours slower is Pacific standard time than Mountain standard time?

6. How many hours slower is Pacific standard time than Eastern standard time?

7. Name the meridians in order that determine the four sections, beginning at the east.

8. Name the four sections in order, beginning with the Eastern section.

9. When it is 11 A.M. at Boston, Eastern standard time, what is the clock (standard) time at San Francisco?

10. When it is 1 P.M. at Chicago, Central standard time, what is the clock (standard) time at San Francisco?

11. When it is 5 P.M. at Denver, Mountain standard time, what is the clock time at San Francisco?

12. When it is 2 A.M. at San Francisco, what is the clock time at New York, Eastern standard time?

13. When it is 12 M. at Santa Fe, Mountain standard time, what is the clock time at Boston?

14. When it is 3.30 P.M., Central standard time, what is the time in the Eastern section?

15. When it is 6.30 P.M., Mountain standard time, what is the time of the Central section?

16. When it is 11.30 A.M., clock time, at Boston, what time is it at Chicago? at Denver? at Sacramento? (Refer to a map.)

17. When it is 6 A.M. at St. Louis, what is the time at New York? at Santa Fe? at Sacramento?

18. If a man whose watch is set by a Boston timepiece travels to San Francisco without resetting his watch, what time will his watch indicate at 7 A.M. in San Francisco?

19. If a man whose watch is set by a San Francisco timepiece travels to Boston without resetting his watch, what time will his watch indicate at 7 A.M. in Boston?

PROMISCUOUS EXAMPLES

1. Divide \$275 between A and B in the ratio of 2 to 3. This means that A is to receive \$2 as often as B receives \$3.

2. Divide 219 A. of land between two brothers in the ratio of 4 : 7.

3. \$21,516 were given to two men in the ratio of 7 to 15. What was each one's share?

4. Divide \$2233 profit between two partners whose investments in the business bear the ratio $\frac{1}{2} : \frac{2}{3}$.

Change the fractions to fractions having a common denominator. The numerators will then have the same ratio as the fractions. Use these numerators as the terms of the ratio.

5. Divide 742 lb. into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

6. What number has the same ratio to 18 that 25 has to 75?

7. The ratio of 7 to $\frac{1}{3}$ equals the ratio of 42 to what number?

8. If 7 men did a certain job in 21 da., how long would it have taken 3 men to do it?

9. How long would it have taken 3 men to do the work that 24 men did in 23 da.?

10. 16 men worked 4 da. of 10 hr. each. How many days of 8 hr. each would it have taken 4 men to do the same work?

11. A cistern can be filled by one pipe in 10 hr. and by another in 15 hr. In what time can the cistern be filled by both running together?

SUGGESTION. — What fraction of the cistern can be filled by *each* in 1 hr.? by both together in 1 hr.?

12. One pipe fills a tub in 12 min., while another requires 15 min. If both are open at the same time, in what time can they fill it?

13. A can do a piece of work in 6 hr., while B would require 8 hr. How long would it take both of them to do the work?

14. James and John are picking berries. James picks 3 qt. an hour and John picks 4 qt. an hour. How long will it take them to pick 56 qt.?

15. Jackson and Johnson start at the same time to walk to Cincinnati, a distance of 28 mi. Jackson walks 4 mi. an hour and Johnson walks $3\frac{1}{2}$ mi. an hour. How long will Jackson have to wait for Johnson at Cincinnati?

16. I have 1 A. of land in the form of a square. Find the length of one side to within .1 of a foot.

17. A man traveled east 81 mi., thence south 108 mi. How far was he from the point of starting?

18. A train goes a mile in 72 sec. How many miles an hour is that?

19. A rectangular field is 400 rd. long and 300 rd. wide. How long a fence must be built to divide it into two equal triangular fields?

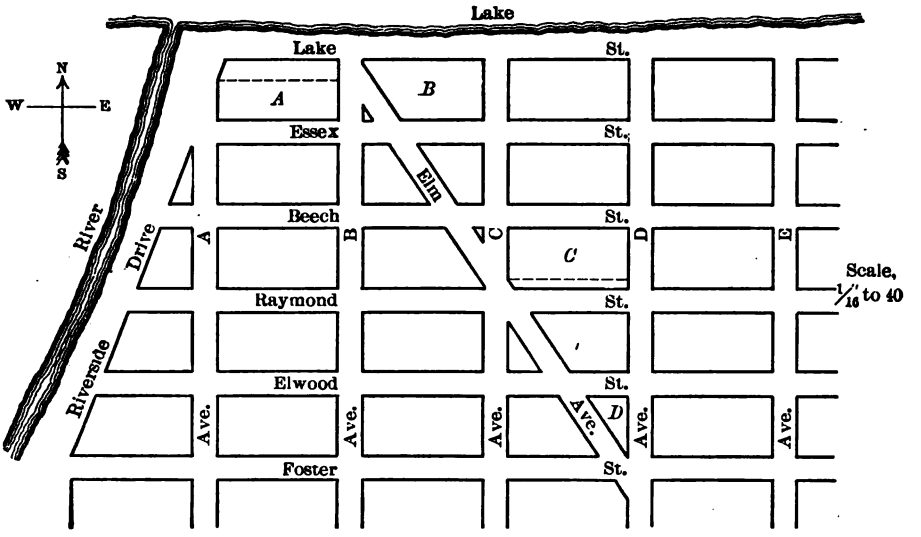
20. A rectangular field has a diagonal 85 rd. long. One of its sides is 68 rd. How many rods of fence are required to inclose the field?

21. Mr. Johnson is to pave Essex St. from Ave. A to Ave. E. How many feet in length is that?

22. Essex St. is 80 ft. wide, but the sidewalk on either side takes up 10 ft. of this width, leaving only 60 ft. in width to be paved. Each paving stone, with the necessary asphalt and other filling, occupies a surface of 4 in. by 6 in. How many paving stones to the square foot?

23. How many paving stones to 1 ft. in length of the street?

24. How many paving stones will be needed for the whole distance?



25. If the stone costs $12 per M delivered and $6 per M for laying, what will it cost Mr. Johnson to pave the street?

26. How many square feet in block A? Use the scale, and note the shape of the northern part of A.

27. How many square feet in block B?

28. How many square feet in triangle D?

29. Colonel Howe has the contract for putting an asphalt pavement on Ave. E from Foster St. to Lake St. The avenue is 80 ft. wide, including the two sidewalks, each of which is 10 ft. wide. It costs the Colonel 75¢ a load (1 cu. yd.) to remove the earth to a depth of 8 in. It then costs him $1.50 per square yard to put down the asphalt pavement. What profit does Colonel Howe make, if his contract allows him $18,000 for the job?

30. What will be the cost of the stone for new sidewalks 10 ft. wide on Ave. E from Foster St. to Lake St. at 10¢ a square foot?

SUGGESTION. — The sidewalk runs 10 ft. out into each street crossed by the avenue, and 10 ft. into Lake St. and Foster St.

31. Find the cost of sidewalks on Foster St. from Riverside Drive to Ave. E, each walk to be 10 ft. wide and to cost 10¢ per square foot. This walk runs 10 ft. into Ave. E and 10 ft. into Riverside Drive, and also 10 ft. out into each avenue which it crosses.

32. A room 18 ft. long, 14 ft. wide, and 10 ft. high is to be lathed and plastered and to have a new floor put in. (*a*) Draw a development of the room. (*b*) What will the lathing cost at 6¢ a square yard? (Ceiling and walls.) (*c*) What will the flooring cost at $30 per M feet, board measure, the floor being 1 in. thick?

33. If poles are placed 10 ft. apart for 50 ft., a pole being at either end, how many poles will there be?

34. How many telephone poles will be needed for a line 5 mi. long, the poles to be 200 ft. apart and a pole at either end of the line?

35. Frank's father gave him a plot of ground on which to plant potatoes. Frank paid 20¢ for a peck of potatoes, $1 for plowing and fitting the ground, and 50¢ for some fertilizer. He hoed and cultivated the crop himself. He raised and sold 5 bu., receiving 75¢ a bushel. How much was his profit?

36. Coal weighs about 50 lb. to the cubic foot. How many pounds of coal in a bin 6 ft. long, 5 ft. wide, and 4 ft. deep? How many tons?

37. At $5.40 a ton, what is the value of the coal in a bin 7 ft. long, 4 ft. wide, and 4 ft. 6 in. deep?

38. A drover bought 20 head of cattle at an average of $30 per head. He paid $2 a head to get them to market. Then 2 of them died. He sold the rest so as to gain 5% on his total investment. At what price per head did he sell? (Answer to the nearest cent.)

39. In 1904 an orchard produced 860 baskets of peaches. That crop exceeded the crop of 1903 by $7\frac{1}{2}\%$. Find the yield in 1903.

40. A farmer paid $1 a bushel for 3 bu. of buckwheat. The tax and interest on the land amounted to $15. The cost of cultivation was $14. He sold his crop of 60 bu. at 80¢ a bushel. What per cent profit did he make?

41. A dealer bought 120 bbl. of apples at $1.50 a barrel. 5 bbl. decayed. He sold the rest at $1.80 a barrel. What per cent profit did he make?

42. A man sold a horse for $240, and made $33\frac{1}{3}\%$. What had the horse cost him?

43. Mr. Mansfield gave me his note for $700, without interest, dated July 2, 1902, and payable 90 da. after date. On Aug. 8, 1902, I presented it at my bank for discount. How much money did my bank place to my credit, the rate of discount being 6%?

44. Mr. Jerome owed his neighbor $675. He gave in payment a note for $300 due 60 da. after date without interest, which had already been running 42 da., a cow worth $60, 15 sheep worth $9 apiece, and the rest in cash. How much cash did he pay, if the rate of discount on the note was 6%?

45. A man bought a house and lot for $6800. He sold the property after 10 yr. for $7000. During the term of his occupancy he paid on the average $60 a year for taxes, $15 a year for insurance, and $50 a year for repairs. If he had rented the house for $600 a year instead of buying it, and invested the $6800 where it would have brought him 5% interest, how much less money would he have on hand?

46. A farmer shipped 400 bbl. of apples and 250 bbl. of potatoes to a commission merchant, who sold the apples for $1.90 a barrel and the potatoes for $2.10 a barrel. He paid for cartage and freight 12¢ a barrel, and charged the farmer $2\frac{1}{2}\%$ commission on the selling price for his services. How much did the farmer receive for his apples and potatoes?

47. A farmer shipped 5 carloads of hay, each containing 10 T., to a dealer. The freight was $2.40 a ton and the dealer's commission $1 a ton. The hay brought $17.50 a ton. How much did the farmer receive?

48. In 1890 the population of a town was 6800. In 1900 the population was 7310. At the same rate per cent of increase a town containing 12,400 people in 1890 should have how large a population in 1900?

49. A peanut vender buys his nuts at $2.50 a bushel and sells them at 10¢ a quart. Find his per cent profit.

50. A boy bought 300 marbles for 50¢ and sold them at the rate of 5 for 1¢. What was his per cent profit?

51. On a bill of goods amounting to $3800 an allowance was made of 20, 10, and 5 off. How much money is necessary to pay the bill?

52. How much money would pay a bill of $3800, a discount of 35% being allowed?

53. A newsboy buys 1-cent papers at the rate of 3 for a cent. What does he make on 300 papers?

54. I bought 7 books at the rate of $10 a dozen, with a discount of 10% for cash. What did the books cost me? (Answer to the nearest cent.)

55. I borrowed $1020 for 2 yr. 8 mo. at 5%. How much must I pay at the end of the time named?

56. A boy on his fifteenth birthday received $100, which he put in the savings bank at once. If the interest is compounded semiannually at 4% per annum, how much money will he have to his credit on his 21st birthday?

57. A gentlemen's club appointed a committee to buy 40 doz. chairs for its new club house. The committee, after investigation, reported the following offers: One firm offered to furnish the chairs for $14 a dozen cash, another firm offered to furnish chairs of the same quality for $16.50 a dozen, with 10 and 5 off for cash. Which was the better offer, and how much better?

58. A coal dealer bought $6000 worth of coal on 6 months' credit, or 5% off for cash. By borrowing the necessary money at 6%, how much can he save by paying cash?

59. If there are $2\frac{1}{4}$ qt. of salt in $5\frac{1}{16}$ gal. of sea water, how many bushels of salt are there in 9000 gal. of sea water? (Use cancellation.)

60. In a certain school there are 330 boys. If the girls are 45% of the school, how many girls are there?

61. A farmer bought a horse for $180, giving his note payable in 1 yr. without interest. At the end of 6 mo. he paid the note, being allowed a discount of 7% a year. How much did he pay?

62. A merchant buys on 60 days' credit a bill of goods invoiced at $1250. Being allowed 5% for cash, he borrows the money at 6% and pays the bill. How much does he save?

63. If 8 men can build 100 panels of fence in 3 hr., in what time can 10 men build 210 panels?

64. A young man aged 18 yr. 6 mo. put $250 in a savings bank. What sum of money will he have in this bank to his credit when he becomes 21 yr. old, the rate of interest being 4%, compounded semiannually?

65. Last year a farmer sold 1440 bu. of corn at 84¢ a bushel. This year he sold 25% more corn at $22\frac{1}{2}$% less per bushel. How much less money did he receive this year than last year?

66. A shoe dealer sold shoes for $3.50 a pair and gained 50¢. What per cent did he gain?

67. A, B, and C cut cord wood. A cut $2\frac{1}{2}$ cords a day, B cut $2\frac{3}{4}$ cords a day. In 6 da. they cut 45 cords. How many cords a day did C cut?

68. If there be a stalk of corn to every 8 sq. ft., and each stalk produce an ear of corn, and 100 ears of corn make a bushel, how many bushels of corn will an acre produce?

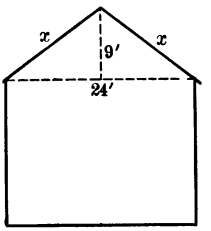
69. Two ships sailing 10 and 12 mi. per hour respectively cross each other's paths at 10 A.M. If one sails due west and the other due north, how far apart will they be at 1 P.M. of the same day? (Answer to 2 decimal places.)

70. How many rods of fence wire will be required for a fence of 4 wires around a rectangular 20-A. field twice as long as it is broad?

SUGGESTION. — By considering the field as half its present length it would be a square field with half its present area, one side of which can be easily found.

71. The inner dimensions of a trough are $1\frac{1}{2}$ ft., $1\frac{3}{4}$ ft., and $5\frac{1}{2}$ in. How many horses, each drinking $2\frac{1}{2}$ gal., could drink one troughful. (See reference tables for the number of cubic inches in a gallon, then use cancellation.)

72. A house with a gable roof is 24 ft. wide. The roof has a 9-ft. rise. How long are the rafters, if they project 1 ft. beyond the eaves?



73. A train 384 ft. long running 6 mi. an hour crosses a bridge. It is 3 min. from the time the locomotive reaches the bridge till the last car leaves the bridge. How long is the bridge?

74. If 2 T. of hay last 30 horses a week, how many tons are needed for 5 horses for 12 wk.?

75. A box without a lid is made of 2-in. plank. How many cubic inches of material are required, the outside dimensions being 2 ft. by 1 ft. 8 in. by 8 in.?

76. A fence incloses a square garden containing 14,400 sq. yd. What will it cost to make a walk 3 ft. wide around the garden just within the fence, at 25¢ a square yard?

77. A room 18 ft. square is carpeted with carpet 27 in. wide. If there is a waste of 9 in. to each strip after the first one for matching, how many yards are required?

78. Mr. Holmes borrowed $400 and, after keeping the money 3 yr. 8 mo. 18 da., returned it with interest at 5%. How much money did he pay?

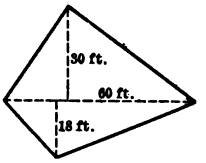
79. April 7, 1900, I gave my note payable on demand for $875. On Dec. 7, 1900, I paid on this note $300. On June 21, 1901, I made another payment of $500. How much money will pay the rest of this note on Jan. 28, 1902, the rate of interest being 6%?

80. How many feet, board measure, will be required for the double floor of a room 18 ft. by 24 ft.? Each floor is 1 in. thick.

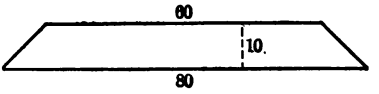
81. How many feet, board measure, in a plank 18 ft. long, 10 in. wide, and $2\frac{1}{2}$ in. thick?

82. How much must be paid for 30 boards, each 16 ft. long, 8 in. wide, and $\frac{1}{2}$ in. thick, at \$20 per M?

83. One diagonal of a trapezium is 60 ft. The perpendiculars from the other two vertices to this diagonal are 30 ft. and 18 ft. respectively. What is the area of the trapezium?



84. A transverse section of a certain railroad bed was 80 ft. long at the bottom, 60 ft. long at the top, and the altitude was 10 ft. How many loads (cubic yards) of earth were required for this roadbed for a distance of 1000 ft.?



85. A certain gable roof is 20 ft. wide on each slope and 60 ft. long. How much will the slate shingles for this roof cost at \$10 a square? (A square = 100 sq. ft.)

86. What will be the cost of plastering the walls and ceiling of a room 12 ft. by 16 ft. by 9 ft., allowing for 6 windows each 2 ft. by 6 ft., and 2 doors each $2\frac{1}{2}$ ft. by 6 ft., the cost per square yard being 35¢?

87. A gable roof 40 ft. long is 12 ft. wide on each slope. How many shingles 4 in. wide laid 4 in. to the weather will be required to shingle this roof, the last row, on either side, being double?

88. A rectangular lot 16 rd. by 32 rd. is to be fenced 6 ft. high. The posts costing 10¢ each are to be set $8\frac{1}{4}$ ft. between centers. The boards are 1-in. boards and cost \$21 per M, board feet. Find the cost of the posts and boards.

89. A carpenter put up the fence in the 88th example, charging 10¢ a rod. How much was his bill?

90. Find the cost of 60 planks, each 14 ft. long, 8 in. wide, and 2 in. thick, at \$22 per M.

91. What is the volume of a round lead pencil $\frac{5}{16}$ in. in diameter and 7 in. long?

92. How many cubic inches of water will a pipe hold that is 1 in. in diameter, interior measurement, and 8 ft. 4 in. long?

93. A cubic foot is equivalent to about $\frac{4}{5}$ bu. How many bushels will a bin hold that is 8 ft. long, 5 ft. wide, and 4 ft. high?

94. A farmer had a field inclosed by three straight sides, 30 rd., 40 rd., and 50 rd. long respectively. How many acres in the field?

NOTE. — What kind of a triangle is this? See if you can do this example differently from the next?

95. A farmer had a field bounded by three straight sides, each 40, 50, and 60 rd. long respectively. How many acres in it? (To 2 decimal places.)

96. How many lead pencils $\frac{5}{16}$ in. in diameter and 7 in. long can be made from a piece of cedar wood 7 ft. long and 5 in. square, allowing $\frac{1}{4}$ of the material for waste?

97. One side of a triangular grass plot is 120 ft. The distance of this side from the opposite corner is 50 ft. How many square yards in the plot?

98. How many revolutions must a wheel make in going 1 mi., if it is 4 ft. high? (The approximate mixed number may be used.)

99. How many cubic feet in a round log of wood 16 ft. long and 2 ft. in diameter.

100. How many square feet in the convex surface of a round log 16 ft. long and 6 ft. in circumference?

101. I have a rectangular court 24 ft. by 40 ft. How many flagstones, each 2 ft. square, do I need to cover it?

102. Find the convex surface of a quadrangular pyramid whose base is 30 ft. square and whose slant height is 320 ft.

103. How much money will pay for the following purchases?

$3\frac{3}{4}$ yd. silk at \$2.40;
20 yd. sheeting at 8¢;
4 pr. blankets at \$5.50;
4 pillows at \$1.75;
10% off to the trade and 3% for cash.

Find the proceeds of a note for:

104. \$90, 2 mo., 6%.

105. \$360, 90 da., 4%.

106. \$875, 30 da., 6%.

107. \$970, 60 da., 5%.

108. A dealer imported 28,000 lb. sugar at $3\frac{1}{8}¢$, paying duties at 40% ad valorem and $\frac{1}{8}¢$ per pound specific. How much did the sugar cost him?

109. A clerk sold \$280 worth of goods in a week. If his salary is \$3.50 a week, and he receives $2\frac{1}{2}\%$ commission on the goods he sells, how much was his income that week?

110. Make a receipted bill for the following: James Decker bought of Joseph Herron, 25 lb. sugar at $5¢$, $\frac{3}{4}$ lb. coffee at $32¢$, $\frac{1}{4}$ lb. tea at $66¢$, 1 doz. eggs at $28¢$.

111. What is the cost of making an excavation 75 ft. × 40 ft. × 6 ft. at \$.75 a load? (1 load = 1 cu. yd.)

112. How many loads of earth must be excavated in digging a circular well 10 ft. in diameter and 16 ft. deep?

113. What is the cost of a stick of timber 30 ft. long and 12 in. square at \$15 per M, board measure?

114. Simplify $\frac{1}{3}$ of $\frac{3\frac{1}{7}}{4}$ of $2\frac{1}{3} \times 14$. (What does $\frac{3\frac{1}{7}}{4}$ mean?)

115. Find the amount at simple interest of \$760.45 for 2 yr. 1 mo. 19 da. at $3\frac{1}{2}\%$.

116. A salesman makes the following sales during one week: \$176.40, \$140.70, \$230, \$85, \$98.40, and \$268.40. If he receives $2\frac{3}{4}\%$ on his sales, what is his income for the week?

117. Find the cost of a pile of wood cut in 4-ft. lengths, piled 6 ft. high. The pile is 40 ft. long and is valued at \$5.20 per cord.

118. Write a note for $125 payable in 90 da. and discount it on the day of issue at 6%.

119. Write a check for $25.40 payable by yourself to James Hutchinson.

120. Oats are what per cent heavier than bran? (See Tables for Reference.)

121. Wheat is what per cent heavier than buckwheat?

122. Buckwheat is what per cent lighter than wheat?

123. A house valued at $11,200 is insured for $\frac{3}{4}$ of its value at $\frac{1}{2}$ of 1% a year. How much is the annual premium?

124. 3 men hire a pasture for $60. A pastures 5 cows, B pastures 3 cows, and C pastures 4 cows. How much ought each to pay?

125. Find the gain per cent on hats bought at $20 a dozen and sold at $2.10 apiece.

126. A cistern is 6 ft. square. How deep must it be to hold 2160 gal. ($7\frac{1}{2}$ gal. to the cubic foot.)

127. Find the value of a rectangular field 50 rd. by 80 rd. at $42.50 per acre.

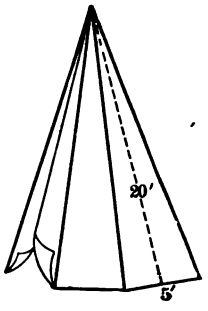
128. What is the ratio of 18 men to 4 men? If 18 men can pave a certain street in 20 da., how long would it take 4 men?

129. What is the ratio of $3\frac{1}{2}$ to 14? If $3\frac{1}{2}$ T. of coal cost $19.50, what will 14 T. cost?

130. When bread was 7¢ a loaf, flour was $7.14 per barrel. What price per barrel for flour would accord with a 6¢ loaf of the same size?

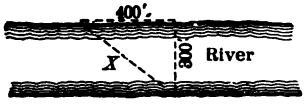
131. About how many gallons will a rectangular tank hold that is $8\frac{1}{4}' \times 4\frac{1}{2}' \times 3\frac{1}{3}'$? (Use the approximate number of gallons to the cubic foot and solve by cancellation.)

132. A hexagonal (6 sided) tent has a slant height of 20 ft., and a base whose sides measure 5 ft. each. How many yards of canvas are needed to make the tent?

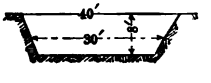


133. Five workmen received $292.50. They worked 20, 24, 30, 25, and 18 da. respectively. As they all received the same wages per day, how much was the total wages of each?

134. A man rowing across a river 300 ft. wide in a freshet was carried downstream 400 ft. by the current. How far was the point of landing from the point of starting?

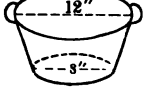


135. A canal 1 mi. long is to be built. A cross section is here given. How many loads of earth must be excavated?



136. How many mowing machines at $62\frac{1}{2}$ can be bought for $750?

137. The bottom of this pan is 8 in. in diameter. The top is 12 in. in diameter. How many square inches of tin are used in making the pan, not counting the waste?



138. How many cubic feet in a ball 1 ft. in diameter? Iron weighs $7\frac{1}{2}$ times as much as water. 1 cu. ft. of water weighs $62\frac{1}{2}$ lb. Find the weight of an iron ball 1 ft. in diameter.

139. A certain roof is in the form of a hemisphere 60 ft. in diameter. Find the cost of painting this roof at 75¢ per square yard.

140. In how many days and hours will a slow steamer going 12 mi. an hour cross the Atlantic ocean — 3000 mi.?

141. The sugar beet yields about 6 % of its weight in sugar. If an acre produces 18 T. of beets, how many acres of beets must be planted to produce 554,000 lb. of sugar?

142. A dealer bought \$1500 worth of goods, which he marked 42 % above cost. He sold them at "bargain sales" for $12\frac{1}{2}$ % less than the marked price. What was his gain?

Simplify:

143. $\dfrac{3\frac{2}{11}}{3\frac{3}{4}}$.

144. $\dfrac{\frac{2}{3} \text{ of } \frac{9}{10}}{\frac{1}{3} \text{ of } \frac{9}{10}}$.

145. $\dfrac{\frac{3}{7} \text{ of } 2\frac{1}{17}}{2\frac{1}{3} \div 2\frac{1}{2}}$.

146. $8 \div \dfrac{1}{4 - \frac{1}{4}}$.

147. $\dfrac{\frac{1}{7}}{8} \times \dfrac{3\frac{1}{2}}{2\frac{1}{3}} \div \dfrac{\frac{1}{2}}{\frac{1}{4}}$.

148. $\dfrac{2\frac{11}{12}}{\frac{4}{7}} \times \dfrac{5\frac{1}{2}}{2\frac{3}{4}}$.

149. A merchant buys silk at \$2.40 a yard, and sells it at \$2.80 a yard. What per cent was his profit?

150. A merchant bought silk at \$2.80 a yard, and some of it being damaged, he sold it at \$2.40 a yard. What per cent was his loss?

151. A boy bought a bushel of nuts for \$1.06 and retailed them at 8¢ a quart. What did he gain per quart? What was his per cent profit on 1 qt.? How much did he gain on the bushel? What was his per cent profit on the bushel?

152. 1 doz. eggs were bought for 30¢ and sold for 35¢. Find the per cent profit? What would be the per cent profit on 2 doz.? on 5 doz.?

153. Find the least common multiple of 25, 26, and 27.

154. Find the least common multiple of 28, 35, and 45.

155. Find the sum of $\frac{31\frac{1}{4}}{\frac{1}{2}}$ and $\frac{7}{\frac{14}{27}}$.

156. A shrub 4 ft. high grew to the height of 4 ft. 8 in. in one year. Find the per cent increase in height.

157. John is 12 yr. old and James is 15 yr. old. James is what per cent older than John? John is what per cent younger than James?

158. In 6 yr. John will be what per cent older than James?

159. The area of Maine is 33,040 sq. mi., and that of Rhode Island is 1250 sq. mi. How many Rhode Islands would equal Maine in area? Give your answer to within .1.

160. How many cubic inches fall on a square foot of ground during a rainfall of 1 in? (A rainfall of 1 in. means a rainfall 1 in. in depth.)

161. How many gallons of water fell on a lot 25 ft. by 100 ft. during a rainfall of 1 in.? (Use the exact number of cubic inches to the gallon. See Tables for Reference.)

162. The population of the United States in 1800 was 5,300,000. In 1900 it was 76,300,000. Find the rate per cent of increase in the 100 yr.

163. Shoes were sold at $\$2.97$ a pair after being marked down 10%. What had been the price before?

164. A boy spelled correctly 42 words of the 50 given him. What per cent of the words did he spell correctly?

165. His seatmate spelled $14\frac{2}{7}\%$ more words correctly. What per cent of his 50 words did his seatmate spell correctly?

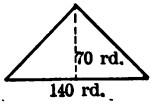
166. $29\frac{3}{4}$ is what per cent of $178\frac{1}{2}$?

167. Find the cube root of 12,167.

168. A train went $134\frac{2}{5}$ mi. in $3\frac{1}{5}$ hr. At what rate is that per hour?

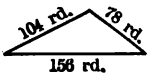
169. I paid $\$22\frac{2}{5}$ for apples costing $\$1\frac{3}{5}$ a barrel, and sold them at $\$2\frac{2}{5}$ a barrel. How much did I gain? What was my per cent gain?

170. How many acres in a triangular piece of land whose base is 140 rd. and whose altitude is 70 rd.? (Use cancellation when possible.)



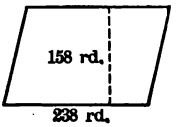
171. How many acres in a triangular piece of land whose base is 95 rd. and whose altitude is 112 rd.?

172. How many acres in a triangular piece of land whose sides are 156 rd., 104 rd., and 78 rd., respectively?



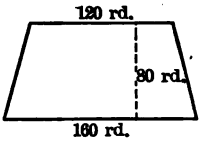
173. How many acres in a triangular piece of land whose sides are 112 rd., 120 rd., and 125 rd., respectively?

174. How many acres in a parallelogram whose base is 238 rd. and whose altitude is 84 rd.?



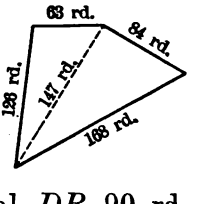
175. How many acres in a parallelogram whose base is 350 rd. and whose altitude is 84 rd.?

176. How many acres in a trapezoid whose bases are 120 rd. and 160 rd. respectively, and whose altitude is 80 rd.?

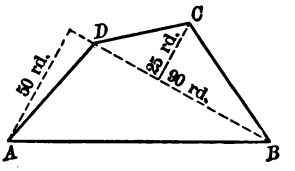


177. How many acres in a trapezoid whose bases are 175 rd. and 180 rd. respectively, and whose altitude is 280 rd.?

178. How many acres in a trapezium whose sides measure 63 rd., 84 rd., 168 rd., and 126 rd., respectively, and whose diagonal shown in the figure measures 147 rd.?



179. A trapezium $ABCD$ has a diagonal DB 90 rd. long, and perpendiculars from the vertices C and A respectively to DB 25 rd. and 50 rd., respectively. How many acres in the trapezium?

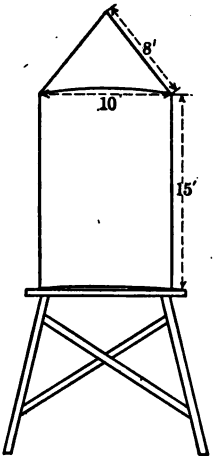


180. A trapezium $ABCD$ has a diagonal DB 80 rd. long, and perpendiculars from the vertices C and D respectively to DB 60 rd. long and 40 rd., respectively. How many acres in the trapezium?

181. The circumference of a circular race track is 1 mi. Find the diameter of the track in rods. Find the area of the track in acres. (Answer to two decimal places.)

182. The circumference of a circular race track is $\frac{1}{2}$ mi. Find the diameter in rods. Find the area in acres.

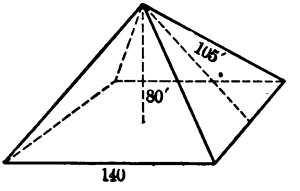
183. A water tank, circular in shape and of uniform diameter, is 15 ft. high and 10 ft. in diameter. How many gallons of water will it hold? (Use the approximate ratio, viz. $7\frac{1}{2}$ gal. to the cubic foot.)



184. The conical roof of the tank is 10 ft. in diameter at the base, and has a slant height of 8 ft. What will it cost to paint this roof at 7¢ a square yard?

185. What will it cost to paint the tank, exclusive of the roof, at 7¢ a square yard?

186. Each of the four sides of a quadrangular pyramid is 140 ft., and the slant height is 105 ft. How large a surface is exposed to the weather?



187. How many cubic feet of stone in this pyramid?

188. A circular well is dug with a uniform diameter to the depth of 15 ft.; then it is dug 5 ft. deeper, tapering to a point in the center like an inverted cone. How many gallons will it hold?

A few examples based upon official records are here given. The pupil will observe that the simple principles with which he has become familiar in exercises with small numbers apply equally well to large numbers.

189. In the year 1900 there were 44,591,851 volumes in 5383 of the public, society, and school libraries in the United States. Find the average number of books in each library. (To the nearest tenth.)

190. In 1875 there were 11,487,778 volumes in the various libraries denoted above. From 1875 to 1880, 1,223,715 volumes were added; from 1880 to 1885, 6,689,706 volumes; from 1885 to 1890, 6,576,444 volumes; from 1890 to 1895, 7,074,229 volumes; and from 1895 to 1900, 11,539,979 volumes. How many volumes, then, in 1900? Compare your answer with the statement in **189**.

191. Find the rate per cent of increase in the number of volumes in 1875 to the number of volumes in 1900. (To the nearest tenth.)

192. During which period was the greatest increase? The least increase? The increase in 1895 to 1900 is what per cent greater than the increase from 1875 to 1880?

193. The U. S. Military Academy Library at West Point had 44,524 volumes in 1900. The 1513 volumes added during 1900 was what per cent of the total? (To the nearest tenth.)

194. For the year ending June 30, 1897, Colorado paid for teachers' salaries $1,385,265.63. That year Arapahoe county paid $470,212.51 for teachers' salaries. Find to the nearest tenth what per cent the salaries paid in that county was of the salaries paid in the whole state.

195. The same state paid for fuel, rent, insurance, and current expenses for public schools for the same year $424,606.11. Find to within .1 of 1% what per cent this sum is of the sum paid for teachers' salaries.

196. The same year Colorado paid for library purposes $26,396.18, of which sum Arapahoe county paid $21,548.57. What per cent of the sum paid by the state was the sum paid by the county? Answer to .1 of 1%.

197. Certain 8 cities in New Jersey had a population in 1895 of 44,500. In their public schools there were 7139 pupils, of whom 3580 were boys and 3550 were girls. The number of boys was what per cent more than the number of girls? Answer to within .1 of 1%.

198. The number of pupils in the public schools of these 8 cities was what per cent of the population?

199. In these 8 cities there were 1631 pupils attending private schools. What per cent of the population was the number of children in private schools?

200. In 1870 there were 53 normal schools in the United States as reported to the Commissioner of Education. In 1879 there were 207 such schools. What was the per cent of increase to the nearest tenth?

201. What per cent of the population was the total number of children attending both public and private schools?

202. In these normal schools there were 178 instructors in 1870. There were 1422 instructors in 1879. Find the per cent of increase to the nearest tenth.

203. There were 10,028 students in these normal schools in 1870, while in 1879 there were 40,029 pupils in these schools. What was the rate per cent of increase to the nearest tenth?

TABLES FOR REFERENCE

Linear Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	= 1 rod (rd.)
320 rods	= 1 mile (mi.)

A hand, 4 in., is used in measuring the height of horses.
A knot, 6086 ft., is used at sea.
A fathom, 6 ft., is used in sounding depths at sea.

Surveyor's Linear Measure

7.92 inches (in.)	= 1 link (li.)
100 links	= 1 chain (ch.)
80 chains	= 1 mile (mi.)

1 ch. = 4 rd. = 22 yd. = 66 ft.

Square or Surface Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

10 sq. ch. = 1 A.; 1 A. = 43,560 sq. ft.

Cubic or Volume Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

1 cu. yd. of earth is 1 *load*.

Surveyor's Square Measure

10 square chains = 1 acre
640 acres = 1 square mile
36 square miles (6 mi. square) = 1 township

Avoirdupois Weight

This is the weight in general use. It is used in weighing everything except the precious metals and jewels, and medicines when dispensed. Medicines are weighed by this table, however, in wholesale dealings.

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweight = 1 ton (T.)

The *long ton* is used in the United States customhouse and in weighing iron and coal at the mines. A long ton weighs 2240 pounds.

1 pound avoirdupois = 7000 grains.

A majority of the states have adopted the following weights for the bushel:

Barley	48 lb.	Corn, shelled	56 lb.
Beans	60 lb.	Oats	32 lb.
Bran	20 lb.	Potatoes	60 lb.
Buckwheat	48 lb.	Rye	56 lb.
Clover seed	60 lb.	Timothy seed	45 lb.
Corn in the ear	70 lb.	Wheat	60 lb.

In Common Use

56 lb. of butter	= 1 firkin
84 lb. of butter	= 1 tub
196 lb. of flour	= 1 barrel
100 lb. of grain	= 1 cental
100 lb. of dry fish	= 1 quintal
200 lb. of beef or pork	= 1 barrel

Troy Weight

Gold, silver, and jewels are weighed by Troy weight.

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

The Troy pound = 5760 gr. It is $\frac{5760}{7000} = \frac{144}{175}$ as heavy as the avoirdupois pound.

Apothecaries' Weight

Apothecaries and physicians use this table in preparing and prescribing dry medicines.

20 grains (gr.)	= 1 scruple (℈)
3 scruples	= 1 dram (ʒ)
8 drams	= 1 ounce (℥)
12 ounces	= 1 pound (℔)

One apothecaries' pound = one Troy pound = 5760 gr.

Apothecaries' Fluid Measure

Apothecaries and physicians use this measure in preparing and prescribing liquid medicines.

60 minims (m.)	= 1 fluidrachm (f ʒ)
8 fluidrachms	= 1 fluidounce (f ℥)
16 fluidounces	= 1 pint (O.)
8 pints	= 1 gallon (Cong.)

1 minim is about 1 drop.

Time

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days	= 1 common year (yr.)
366 days	= 1 leap year
100 years	= 1 century (cen.)

The Months of the Year

January (Jan.)	31 da.	July	31 da.
February (Feb.)	28 or 29 da.	August (Aug.)	31 da.
March (Mar.)	31 da.	September (Sept.)	30 da.
April (Apr.)	30 da.	October (Oct.)	31 da.
May	31 da.	November (Nov.)	30 da.
June	30 da.	December (Dec.)	31 da.

In most business dealings 30 days are considered as 1 month and 360 days as 1 year.

In recalling the number of days in a month these lines will prove helpful:

"Thirty days hath September,
April, June, and November."

Any year is a leap year if the number of the year is exactly divisible by 4, unless the number of the year ends in two ciphers; in which case it is a leap year if exactly divisible by 400.

Circular or Angular Measure

60 seconds (″)	= 1 minute (′)
60 minutes	= 1 degree (°)
360 degrees	= 1 circumference (C.)

Liquid Measure

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
$31\frac{1}{2}$ gallons	= 1 barrel (bbl.)
2 barrels	= 1 hogshead (hhd.)
1 gallon	= 231 cu. in.

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)
1 bushel = 2150.42 cu. in.

Counting

12 units = 1 dozen (doz.)
12 dozen = 1 gross
12 gross = 1 great gross
20 units = 1 score

Paper

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

United States Money

10 mills (m.) = 1 cent (ct. or ¢)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)
10 dollars = 1 eagle (E.)

United States Coins

Bronze: the cent.
Nickel: the five cent piece.
Silver: the dime, quarter dollar, half dollar, and dollar.
Gold: the quarter eagle, half eagle, eagle, and double eagle.

British or Sterling Money

4 farthings (far.)	= 1 penny (*d.*)
12 pence	= 1 shilling (*s.*)
20 shillings	= 1 pound (£)
5 shillings	= 1 crown
21 shillings	= 1 guinea

The value in United States money of the monetary unit of the following nations is:

Austria-Hungary	Crown	$.203
Brazil	Milreis	.546
Canada	Dollar	1.00
Chili	Peso	.912
Denmark	Crown	.268
France	Franc	.238
Germany	Mark	.238
Great Britain	Pound Sterling	4.8665
Italy	Lira	.193
Japan	Yen	.997
Mexico	Gold Dollar	.983
	Silver Dollar	.528
Russia	Ruble	.772
Spain	Peseta	.193

APPENDIX (Optional)

I. HIGHEST COMMON FACTOR

1. The highest common factor of several numbers is the largest number that will exactly divide each of them.

The highest common factor is also known as the greatest common divisor and the greatest common measure.

2. Find the highest common factor of 24, 36, 72, and 100.

First Method

$24 = 2 \times 2 \times 2 \times 3$
$36 = 2 \times 2 \times 3 \times 3$
$72 = 2 \times 2 \times 2 \times 3 \times 3$
$100 = 2 \times 2 \times 5 \times 5$

H.C.F. $= 2 \times 2 = 4$

Second Method

2	24	36	72	100
2	12	18	36	50
	6	9	18	25

H.C.F. $= 2 \times 2 = 4$

First Method. — 2 is found twice as a prime factor in every number. No other prime factor is found in all of the numbers. $2 \times 2 = 4$. 4 is the H. C. F.

Second Method. — We divide by *any* number that will exactly divide all of the numbers. Divide the quotients in like manner. Continue thus, until the quotients have no common factor greater than one. The product of the several divisors is the highest common factor.

A common divisor of several numbers contains only prime factors that are common to the several numbers.

Find the highest common factor of:

3. 48, 96, 72.
4. 28, 42, 56.
5. 45, 90, 60.
6. 42, 63, 84.
7. 60, 90, 120.
8. 84, 126, 210.
9. 60, 80, 40.
10. 132, 165, 231.
11. 34, 51, 85.
12. 18, 54, 63.

By comparing the two methods, it will be observed that only the common prime factors are selected in the first case and only the common factors are found in the second case.

II. HIGHEST COMMON FACTOR. LARGE NUMBERS

9 exactly divides 117 and 36. It also exactly divides their sum, 153, 17 times. It also exactly divides the sum of 117 and 3 times 36, or 225, 25 times, and the difference between 117 and 3 times 36, or 9, once.

By thus considering any two numbers, as 117 and 36, it will be found that any common divisor of them, as 9, will divide their sum or difference, and also the sum or difference of any multiples of these numbers.

Test the above principle:

1. Numbers 28, 63, divisor 7.

7 divides their sum, or ——, —— times.

7 divides their difference, or ——, —— times.

7 divides the sum of 63 and 2×28, or ——, —— times.

7 divides the difference of 63 and 2×28, or ——, —— times.

2. Numbers 42, 87, divisor 3.

3. Numbers 35, 14, divisor 7.

4. Numbers 22, 55, divisor 11.

5. Numbers 40, 85, divisor 5.

6. Find the highest common factor of 2125 and 10608.

	2125	10608
	2108	8500
H.C.F.	17	2108
		2108

2125 is contained in 10,608 4 times (the quotients may be omitted) with a remainder of 2108. 2108 is contained in 2125 once with a remainder of 17. 17 is contained in 2108 124 times. ∴ 17 is the H. C. F. of 2125 and 1,0608.

To prove this:

First, 17 exactly divides 2108 and hence it exactly divides 2108 + 17, or 2125. 17 exactly divides 2108 and 2125, hence it exactly divides 2108 + 4 × 2125, or 10608. 17 is, therefore, a common factor of 2125 and 10608.

Now, is it the highest common factor? Any number which exactly divides 2125 and 10608 will exactly divide the difference between 10608 and 4 × 2125, or 2108. Any number which will exactly divide 2125 and 2108 will exactly divide the difference between 2125 and 2108, or 17. But the largest number which exactly divides 17 is 17. Hence 17 is the H. C. F. of 2125 and 10608.

Find the H. C. F. of:

7. 688 and 1677.

8. 644 and 759.

9. 1175 and 6775.

10. 1943 and 2177.

11. 2231 and 2813.

12. 221 and 238.

13. 345, 391, and 506.*
14. 377, 435, and 551.*
15. 161, 207, and 299.*
16. 155, 279, and 496.*

A fraction may be reduced to its lowest terms by dividing both terms of the fraction by their highest common factor.

Reduce the following fractions to their lowest terms:

17. $\frac{688}{1677}$.
18. $\frac{644}{759}$.
19. $\frac{1175}{6775}$.
20. $\frac{221}{238}$.
21. $\frac{26}{39}$.
22. $\frac{51}{68}$.
23. $\frac{76}{95}$.
24. $\frac{115}{138}$.
25. $\frac{261}{319}$.
26. $\frac{134}{201}$.
27. $\frac{142}{355}$.
28. $\frac{194}{291}$.
29. $\frac{121}{143}$.
30. $\frac{91}{195}$.
31. $\frac{87}{319}$.
32. $\frac{187}{255}$.

III. CUBE ROOT (*c*). (Continued from Lesson LXII)

1. Find the cube of 75. $75 = 70 + 5$. We may find the cube of 75 in this way:

$$\begin{array}{r} 70 + 5 \\ 70 + 5 \\ \hline (5 \times 70) + 5^2 \\ 70^2 + (5 \times 70) \\ \hline 70^2 + 2(5 \times 70) + 5^2 \\ 70 + 5 \\ \hline (5 \times 70^2) + 2(5^2 \times 70) + 5^3 \\ 70^3 + 2(5 \times 70^2) + (5^2 \times 70) \\ \hline 70^3 + 3(5 \times 70^2) + 3(5^2 \times 70) + 5^3 \end{array}$$

5 is a factor of the last three terms. Hence, we may write the result thus: $70^3 + 5(3 \times 70^2 + 3 \times 5 \times 70 + 5^2)$.

* Find the H. C. F. of any two of the numbers, then find the H. C. F. of the result and the remaining number.

Suppose, now, we wish to find the cube root of 421,875.

70	421′875	70+5
	343 000	
$3 \times 70^2 = 14700$	78 875	
$3 \times 5 \times 70 = 1050$		
$5^2 = 25$		
15775	78 875	

The cube of 70 = 343000. The cube of 80 = 512000. The latter is too large. Hence, 70 is the first part of the cube root. The tens' figure of the cube root is 7.

Now, notice that in the expression, $5(3 \times 70^2 + 3 \times 5 \times 70 + 5^2)$, the number 5 is the second figure in the number 75 that was cubed. Hence, in finding the cube root, the complex factor, $(3 \times 70^2 + 3 \times 5 \times 7 + 5^2)$, will be the true divisor, and 3×70^2 the trial divisor.

Therefore, we square 70 and multiply by 3, obtaining 14700. Dividing 78875 by 14700, we obtain the quotient, 5.

Now, completing the divisor by adding $3 \times 5 \times 70$, and 25, we find the true divisor to be 14700 + 1050 + 25, or 15775. Multiplying 15775 by 5, we obtain 78875. The cube root of 421875 is 75. Evidently, to prove this answer correct, we may cube 75.

Compare with the following:

	421′875	75
	343	
$300 \times 7^2 = 14700$	78 875	
$30 \times 7 \times 5 = 1050$		
$5^2 = 25$		
15775	78 875	

2. Extract the cube root of 122,763,473.

		122′763′473	497
		64	
$300 \times 4^2 =$	4800	58 763	
$30 \times 4 \times 9 =$	1080		
$9^2 =$	81		
	5961	53 649	
$300 \times 49^2 =$	720300	5 114 473	
$30 \times 49 \times 7 =$	10290		
$7^2 =$	49		
	730639	5 114 473	

The cube of 4.97 equals 122.763473. Corresponding to the two figures in the decimal part, .97, we evidently have two groups 763 and 473 in the cube. Therefore, we begin at the decimal point and point off the number into periods of three figures each. In decimals *every* period must have three figures. Ciphers must be annexed where there are not figures enough. When the decimal in the root is not exact, use the sign +.

3. Find the cube root of 9.7 to two decimal places.

		9.700′000	3.02+
		9	
$300 \times 30^2 =$	270000	700 000	
$30 \times 30 \times 2 =$	1800		
$2^2 =$	4		
	271801	543 608	

IV. CUBE ROOT (*d*)

1. $\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$.

2. $\sqrt[3]{\frac{5}{27}} = \frac{\sqrt[3]{5}}{\sqrt[3]{27}} = \frac{1.709}{3} = .569$.

3. $\sqrt[3]{\frac{7}{25}} = \sqrt[3]{.28} = .654$.

Which of the denominators above given is a perfect cube? When the denominator is a perfect cube, extract the cube root of each term, and change the resulting fraction to a decimal if desired.

When is it desirable to change a fraction to a decimal before extracting its cube root? Can you give a reason for this?

Find the cube root of:

4. 19,683.
5. 21,952.
6. 24,389.
7. 27,000.
8. 1,860,867.
9. 15,813,251.
10. 31.255875.
11. 128.024064.
12. .000625.
13. .000000625.
14. .8 (3 decimal places).
15. .27.
16. .1.
17. $\frac{5}{7}$. (See Ex. 3.)
18. $\frac{5}{64}$. (See Ex. 2.)
19. $3\frac{1}{3}$ (3.333′333).

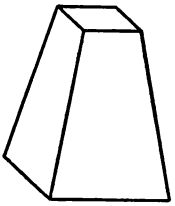
20. A bushel contains 2150.42 cu. in. Find the length of the inside edge of a cubical box that will hold 1 bu. (Answer to 1 decimal place.)

21. A gallon contains 231 cu. in. Find the length of the inside edge of a cubical box that will hold 1 gal. (1 decimal place.)

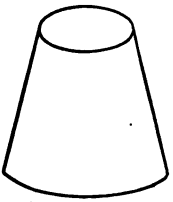
22. The area of the surface of a cube is 1536 sq. in. Find the volume.

V. FRUSTUMS OF PYRAMIDS AND CONES

1. If the top of a cone or a pyramid is cut off by a plane parallel to the base, the part left is called a *frustum*.



Frustum of a Pyramid



Frustum of a Cone

2. The side faces of the frustum of a pyramid are all trapezoids. If the convex surface of either frustum be fitted with paper and the paper be unrolled, it will be a trapezoid. Therefore,

3. To find the convex surface of the frustum of a cone or a pyramid, multiply one half of the sum of the circumferences or the perimeters by the slant height.

4. Find the convex surface of the frustum of a cone whose bases have circumferences of 10 ft. and 8 ft. respectively, and whose slant height is 7 ft.

5. Find the convex surface of the frustum of a pyramid whose lower base is a triangle with sides of 5 ft. each, and whose upper base is a triangle with sides of 3 ft. each, and whose slant height is 4 ft.

6. Find the solid contents of the frustum of a square pyramid, each side of the upper base being 3 ft. and of the lower base 4 ft., and whose altitude is 12 ft.

$3 \times 3 = 9$, area of upper base.

$4 \times 4 = 16$, area of lower base.

$\sqrt{9 \times 16} = 12$, square root of their product.

$9 + 16 + 12 = 37$, sum of the areas and the square roots.

$\frac{1}{3}$ of $12 \times 37 =$ ——, solid contents.

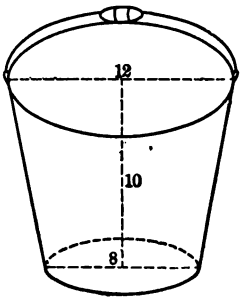
7. To find the solid contents of the frustum of a pyramid or a cone, add the areas of the bases. Find the square root of the product of the bases. Multiply the sum of the two results by one third of the altitude.

8. Find the solid contents of the frustum of a cone whose altitude is 10 ft., and the diameter of whose bases are 4 ft. and 3 ft. respectively.

9. How many cubic feet in 75 round fence posts 6 ft. long, the diameters of the ends being 6 in. and 4 in. respectively?

10. How many cubic feet in 40 fence posts 8 ft. long, one end of each being 6 in. square and the other 4 in. square?

11. This pail is 12 in. in diameter at the top, 8 in. in diameter at the bottom, and 10 in. in height. How many gallons will it hold?



VI. AN ACTUAL EXPERIENCE

The hennery shown above is 45 ft. long and 12 ft. wide. The front is 7 ft. high, and the back 4 ft. high. The roof is 45 ft. long and 14 ft. wide. The windows are 3 ft. by 6 ft. All the windows and the only door are shown. The door is 2 ft. 6 in. by 5 ft. 6 in.

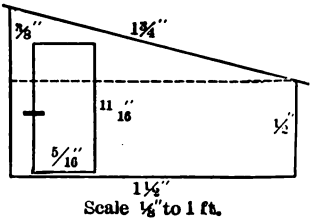
1. How many square feet of window surface is there?

2. What did the windows cost, each window pane costing 7¢, and each of the three window sashes costing $1.50?

3. How many square feet of surface in the front exclusive of the windows?

4. How many square feet of surface in the rear?

5. This drawing shows the end containing the door drawn to the scale of $\frac{1}{8}'' : 1'$. How many square feet of surface in this end exclusive of the door? Note the triangle and the rectangle.

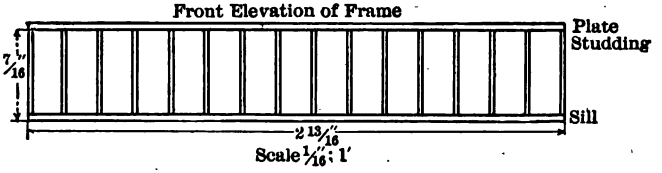


6. What is the area of the other end?

7. The roof is made of hemlock boards costing $20 per M, covered with paroid roofing, 4 rolls at $2.25 per roll. What is the cost of the roof?

8. The siding is all 1 in. thick. Find the cost of the siding at $30 per M.

9. The door cost $2.25. Find the total cost of the siding, windows, door, and roof.



Here is a diagram showing the front of the frame.

10. Note the scale. What is the length of the sill? the plate?

11. The studding consists of 2 × 4 (two by four) stuff. What is the length of each piece? How many feet, board measure, in the studding?

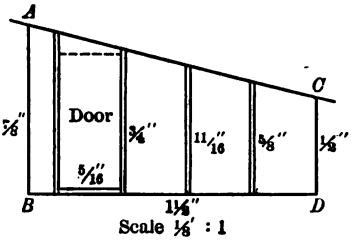
12. The plate and sill are each 3 × 6. How many feet, board measure, in both?

13. The studding in the rear portion of the frame is only 4 ft. long. It is placed 3 ft. between centers, as in the front. (3 ft. between centers means 3 ft. from the center of one piece to the center of the next.) How many feet, board measure, in the studding in the rear portion?

14. The plate and sill in the rear are like the plate and sill in front. How many feet, board measure, in them?

15. The rafters are made of 2×4 scantling, and are each 14 ft. long. They are placed 3 ft. between centers, and are, therefore, the same in number as the studding in front. How many feet, board measure, in the rafters?

The end showing the necessary studding is here drawn to a scale of $\frac{1}{8}$ in. to 1 ft. On account of the desire to have the door near the front, the first 2×4 is placed 1 ft. from the front. How wide is the door? Next is placed the second 2×4. This leaves room for only two more 2×4's, which are approximately 3 ft. between centers. The 2×4 AB at the front corner and the 2×4 CD at the back corner were included when we considered the front and back.



16. The first 2×4 at the left of the door is scant $\frac{7}{8}''$ in the drawing, but $\frac{7}{8}''$ is sufficiently exact for our purpose. How many feet, board measure, in the four 2×4's in this end? The other end is the same. How many feet, board measure, in the 2×4's of both ends?

17. The sill BD is 3×8. The sill at the other end is of the same size. How many feet, board measure, in both sills?

18. At \$22 per M, what is the cost of the rafters, studding, plates, and sills?

19. What is the total cost of the lumber in the structure, exclusive of the interior furnishings, which we have not considered?